

UNIVERSITÉ DE REIMS CHAMPAGNE-ARDENNE
ÉCOLE DOCTORALE MATHÉMATIQUES, PHYSIQUE, SCIENCES DU NUMÉRIQUE ET DE L'INGÉNIEUR (n°620)
CENTRO FEDERAL DE EDUCAÇÃO TECNOLÓGICA DE MINAS GERAIS
PROGRAMA DE PÓS-GRADUAÇÃO EM MODELAGEM MATEMÁTICA E COMPUTACIONAL

TESE EM COTUTELA

Apresentada para obtenção dos graus de

DOUTOR EM MODELAGEM MATEMÁTICA E COMPUTACIONAL
pelo CENTRO FEDERAL DE EDUCAÇÃO TECNOLÓGICA DE MINAS GERAIS e de
DOCTEUR EN AUTOMATIQUE ET TRAITEMENT DU SIGNAL
pela UNIVERSITÉ DE REIMS CHAMPAGNE-ARDENNE

Linha de Pesquisa: SISTEMAS INTELIGENTES

Apresentada e defendida publicamente por

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3 de dezembro de 2025

SAMPLED-DATA CONTROL OF LPV SYSTEMS UNDER MAGNITUDE AND RATE SATURATING ACTUATORS

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(Catalogação - Biblioteca Universitária – Campus Divinópolis – CEFET-MG)

O48s	Oliveira, Lucas Arantes Lemos. Sampled-data control of LPV systems under magnitude and rate saturating actuators. / Lucas Arantes Lemos Oliveira – Belo Horizonte, 2025. 184 p. : il. Orientador: Prof. Dr. Valter J. S. Leite. Coorientador: Prof. Dr. Kevin Guelton Tese (Doutorado) – Centro Federal de Educação Tecnológica de Minas Gerais, Campus Divinópolis, Programa de Pós-Graduação <i>Stricto Sensu</i> em Modelagem Matemática e Computacional, 2025. 1. LPV systems. 2. Sampled-data control. 3. Actuator saturation. 4. Parameter mismatch. 5. Polynomial negative definiteness. I. Leite, Valter J. S. II. Guelton, Kevin. III. Centro Federal de Educação Tecnológica de Minas Gerais. IV. Título. CDU: 681.51
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Bibliotecário Responsável: Leonardo Luis Ribeiro CRB-6:2634



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**“SAMPLED-DATA CONTROL OF LPV SYSTEMS UNDER MAGNITUDE
AND RATE SATURATING ACTUATORS”.**

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*For my father (in memoriam), who must be very proud,
and for my beloved Laura, with love.*

Preface

Dear reader,

A friend once told me that Ph.D. stands for *Pretty Huge Deceptions*. I can't say I fully agree, though I must admit, there were plenty of disappointments along the way.

This journey began the day I first stared at the derivatives of those so-called looped-functionals (you will see them more than once in the pages ahead). Right after that, I called Luiza and cried. At the time, it felt like the end of the world. Now, looking back, I realize that was actually the easy part. Someone had already proposed those functionals, solved them, and probably had a decent night of sleep afterward. The real challenge was still ahead: I had to find something of my own to contribute. And I had no idea what that would be.

Yes, there were frustrations. Many, in fact. Some academic, others... well, like watching Galo (Atlético Mineiro) lose some finals in 2024 and 2025, let's not get into that. But there were also moments of joy: the birth of my niece Laura and the unique experience of living in France for a while.

This thesis is the result of four full years of my life. During this time, I thought about the same topic every single day. No exceptions. Yes, even on Galo match days.

So here it is, the story of these past four years. I hope you enjoy the read as much as I enjoyed living it.

Lucas



The iconic cover of *Clube da Esquina* (1972),
by Milton Nascimento and Lô Borges.
Photograph by Carlos da Silva Assunção
Filho (Cafi).

Com sol e chuva você sonhava
Queria ser melhor depois
Você queria ser o grande herói das estradas
Tudo que você queria ser

Sei um segredo
Você tem medo

Só pensa agora em voltar
Não fala mais na bota e do anel de Zapata
Tudo que você devia ser
Sem medo

Não se lembra mais de mim
Você não quis deixar que eu falasse de tudo
Tudo que você podia ser
Na estrada

Ah! Sol e chuva
Na sua estrada

Mas não importa, não faz mal
Você ainda pensa e é melhor do que nada
Tudo que você consegue ser
Ou nada

Não importa, não faz mal
Você ainda pensa e é melhor do que nada
Tudo que você consegue ser
Ou nada

Tudo O Que Você Podia Ser
Clube da Esquina (1972)
MILTON NASCIMENTO & LÔ BORGES

Acknowledgment

I would like to express my sincere gratitude to the members of the examination committee for their valuable suggestions and careful corrections, which greatly improved this manuscript. I especially thank Prof. Alisson M. da Silva and Prof. Elizabeth F. Wanner for coordinating the PPGMMC and enabling the cotutelle agreement; Prof. João M. Gomes da Silva Jr. and Dr. Sophie Tarbouriech, whose work has guided this research; Professor Pedro L. D. Peres for his kindness, meticulous corrections, and thoughtful suggestions; and Dr. Koffi M. D. Motchon for his contributions and support throughout this work.

I am deeply grateful to Professor Kevin Guelton for the ideas that guided this thesis. Thank you for welcoming me into your home and your country. Remember that you will always have a friend here in Brazil.

I am especially grateful to Professor Valter J. S. Leite for his guidance, trust, and constant support over the years. Our collaboration has been a source of great pride for me, and contributing to his research has been both an honor and a privilege.

I would also like to thank my colleagues from the Signals and Systems Laboratory at CEFET-MG, Marcus, Gabriel, Marcela, and Heros, for the many conversations about science and about life.

To my colleagues at CReSTIC, Alex, Noémie, Álan, and Sara, thank you for making my life in France easier and more enjoyable.

To my sister Lívia and Paulo, for their love, and to little Laura, our piece of love.

To Luiza, thank you for sharing this journey with me and for making me happier along the way.

To my mother, thank you for loving me so deeply.

I gratefully acknowledge the institutional support of the Centro Federal de Educação Tecnológica de Minas Gerais (CEFET-MG) and the Université de Reims Champagne-Ardenne (URCA).

I also acknowledge the financial and institutional support provided by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES), the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), and the Fundação de Amparo à Pesquisa do Estado de Minas Gerais (FAPEMIG).

Finally, I extend my appreciation to all those who, in any way, contributed to the realization of this work.

Contents

List of Figures	vii
List of Tables	ix
List of Acronyms	xi
List of Notations	xiii
1 General Introduction	1
1.1 Objectives	8
1.2 Thesis Outline and Contributions	8
2 Fundamental Concepts	11
2.1 State Feedback Control of Parameter-Dependent Systems	11
2.2 Lyapunov Stability	15
2.2.1 Parameter-Dependent Lyapunov Candidate Function	16
2.3 Input-Delay and Looped-Functionals	18
2.3.1 Stability Conditions as Quadratic Polynomials	23
2.4 Input Saturation	26
2.4.1 Magnitude Saturation	27
2.4.2 Magnitude and Rate Saturation	29
2.5 Final Considerations	33
3 Stabilization of Discrete-Time LPV Systems with Saturating Actuators	35
3.1 Problem Formulation	36
3.2 Preliminary Results	39
3.3 Main Results	42
3.3.1 Optimization Procedures	46
3.4 Illustrative Examples	47
3.4.1 Estimated Region of Attraction of LPV and T-S Fuzzy Models	47
3.4.2 Reference Tracking of an LPV Model	50

3.4.3	Real-Time Control of a <i>quasi</i> -LPV System	52
3.5	Final Considerations	57
4	New Negative Definiteness Conditions for Quadratic Functions in LPV Systems	59
4.1	Recent Results and Problem Formulation	59
4.2	Main Results	62
4.2.1	Homogeneous Polynomial Approach	62
4.2.2	Bézier Curves Approach	63
4.2.3	Computational Complexity	65
4.3	Application to LPV Sampled-Data Control	66
4.4	Illustrative Examples	68
4.4.1	Numerical Example of a Scalar Quadratic Inequality	68
4.4.2	Benchmark of a Sampled-Data Control System	70
4.5	Final Considerations	72
5	Mitigating Parameter Mismatch in LPV Sampled-Data Control	75
5.1	Parameter-Dependent Stabilization of LPV Sampled-Data Systems via Linear Polynomial Condition	76
5.1.1	Looped-Functionals and The Mean Value Approach	77
5.1.2	Stabilization Conditions	79
5.1.3	Illustrative Example	82
5.2	Robust Stabilization of LPV Sampled-Data Systems via Quadratic Polynomial Conditions	84
5.2.1	Second-Order Looped-Functional Conditions	85
5.2.2	Negative Definiteness Lemmas	86
5.2.3	Robust Stabilization Conditions	86
5.2.4	Illustrative Examples	88
5.2.5	<i>Quasi</i> -LPV Real-Time Experimental Validation	90
5.3	Parameter-Dependent Control of LPV Sampled-Data Systems via Quadratic Polynomials	96
5.3.1	Illustrative Example	97
5.4	Final Considerations	98
6	Sampled-Data Control of LPV Systems under Saturating Actuators	99
6.1	Problem Formulation	100
6.2	Auxiliary Results	102
6.3	Main Results	105
6.3.1	Optimization Procedures	109
6.4	Illustrative Examples	109
6.5	Real-Time Experimental Validation with Saturating Actuators	111

6.6	Final Considerations	116
7	Conclusion and Perspectives	117
7.1	Concluding Remarks	117
7.1.1	Future Research	120
7.2	Additional Results Developed Along the Doctoral Research	121
7.2.1	Network Control Systems Design under Deception Attacks with Dynamic Event-Triggering Mechanism	121
7.2.2	Co-Design and Evolutionary Optimization of Periodic Event-Triggered Robust Control for LPV Systems	123
7.2.3	Relaxed Co-Design Conditions for Periodic Event-Triggered Control of Lur'e Systems	125
7.3	Publications	126
7.4	Awards	128
	Bibliography	130
A	Equivalent inequality conditions to the statement of Theorem 4.1	153
	Abstract	155

List of Figures

1.1	Typical architecture of a networked control system.	2
1.2	Schematic overview of the control and design chain for a continuous-time networked nonlinear plant.	3
1.3	Coupled tank system with nonlinear dynamics.	4
1.4	Comparison between the discrete-time model and the experimental data of the tank system with nonlinear dynamics.	5
1.5	Overview of the sampled-data control implementation for a continuous-time nonlinear plant.	6
2.1	Schematic diagram of the sampled-data state feedback control structure.	14
2.2	Time-varying delay as a piecewise linear function $\tau(t) = t - t_k$	20
2.3	Illustration of Theorem 2.3 (Adapted from Seuret (2012)).	22
2.4	Geometrical interpretation of the approach proposed in Kim (2011)	24
2.5	Geometrical interpretation of the lemmas Park and Park (2020) and Liu et al. (2023)	25
2.6	Saturation function.	28
2.7	Diagram of the continuous-time PMSL used to model magnitude and rate saturation for an input ℓ	31
2.8	Step response of the continuous-time PMSL for an input ℓ and different values of λ	31
2.9	Diagram of the discrete-time PMSL used to model magnitude and rate saturation for an input ℓ	32
2.10	Step response of the discrete-time PMSL for an input ℓ and different values of λ	32
3.1	Schematic diagram of the closed-looped system.	37
3.2	Cut of the estimate region of attraction $\mathcal{R}_{\mathcal{E}_a}$	48
3.3	Cut of the estimate region of attraction for $\sigma = 0.999$	50
3.4	Time-behavior of the Lyapunov function for the trajectories.	50
3.5	Outputs of the system tracking the references signals.	53
3.6	Control signals applied to the F-4 fighter aircraft model.	53
3.7	Diagram of the coupled tank system used to perform a real-time experiment.	54

3.8	Real-time reference track experiment performed with the designed controller. . .	57
3.9	Phase plane portrait with experimental data.	58
4.1	Examples of Bézier curves.	64
4.2	Demonstration of de Casteljau's algorithm for a Bézier curve.	65
4.3	Comparison of the feasibility fields obtained from the different approaches with no partitions.	70
4.4	Comparison of the feasibility fields of Lemmas 4.2 and Theorems 4.1-4.3.	71
5.1	Top plot: states of the system, $x(t)$. Bottom plot: random sampling intervals $\eta_k \in [0.001, 1.450]$ s and the sampled-data control signal $u(t_k)$	84
5.2	Top plot: states of the system, $x(t)$, asymptotically converging to the origin. Bottom plot: random sampling intervals $\eta_k \in [0.200, 0.528]$ s, dashed red lines are the bounds of the intervals.	91
5.3	Top plot: time-varying parameters $\alpha(t) \in \Gamma_8$. Bottom plot: control signals during the simulation.	91
5.4	Schematic of the coupled tank system used for real-time experimentation.	92
5.5	State-space of the region of attraction with simulated trajectories.	94
5.6	Real-time experiment with random time sampling interval.	95
5.7	Real-time experiment with constant time sampling interval set at the maximum allowed.	95
5.8	Real-time experiment with increasing time sampling interval.	96
5.9	Real-time experiment with decreasing time sampling interval.	96
6.1	Control topology of sampled-data state feedback with actuators under magnitude and rate saturation.	101
6.2	Cut of the estimate region of attraction $\mathcal{L}_V$ at $x_a = 0$ obtained with optimization procedure $\mathcal{O}_2$ and several trajectories projections.	110
6.3	Top plot: states of the system, $x_s(t)$. Bottom plot: state of the actuator dynamics $\dot{x}_a(t)$ on left scale, state of the actuator $x_a(t)$, and control signal magnitude saturation function $\text{sat}_M(u(t))$ on right.	111
6.4	Cut of the estimate region of attraction $\mathcal{E}(P(\alpha(t)))$ at $x_a = 0$ obtained with optimization procedure $\mathcal{O}_3$, and real-time experiment trajectory (purple line). . .	112
6.5	Real-time experiment under saturating actuator ($\eta_k = \mathcal{T}_2 = 30$ s).	113
6.6	Actuator dynamics, $\dot{x}_a(t)$, real-time experiment with rate saturation ($\eta_k = \mathcal{T}_2 =$ 30 s).	114
6.7	Time-varying parameters during the experiment ($\eta_k = \mathcal{T}_2 = 30$ s).	114
6.8	Real-time experiment under saturating actuator with increasing sampling interval ($\eta_{k+1} = \eta_k + 1$, $\eta_0 = 1$ s).	115
6.9	Actuator dynamics, $\dot{x}_a(t)$, real-time experiment with rate saturation ($\eta_{k+1} =$ $\eta_k + 1$, $\eta_0 = 1$ s).	115

6.10	Time-varying parameters during the experiment ($\eta_{k+1} = \eta_k + 1$, $\eta_0 = 1$ s).	116
7.1	Summary of the main topics and objectives of this doctoral thesis.	118
7.2	Block diagram of the NCS under the proposed dynamic ETM and subject to deception attacks.	122
7.3	Topology of the event-triggered control.	124
7.4	Topology of the event-triggered control for Lur'e systems.	126
7.5	Plaque of recognition for the SBA Award – Doctorate 2024.	129

List of Tables

3.1	Comparison of the estimated areas achieved with optimization procedure $\mathcal{J}_1$. . .	49
4.1	Number of matrix inequality rows, number of additional decision variables and types of negative definiteness conditions in Lemmas 4.1-4.3 and Theorems 4.1-4.3.	66
4.2	Feasibility of the different negative definiteness conditions.	69
4.3	Maximum allowable sampling interval and solver time from Lemmas 4.2-4.3 and Theorems 4.1-4.3.	72
5.1	Maximum sampling interval, $\mathcal{T}_2$, for different bounds on $\dot{\rho}(t)$	83
5.2	Maximum value of $\mathcal{T}_2$ (Ex. 2).	89
5.3	Maximum value of $\mathcal{T}_2$	98

List of Acronyms

BMI	Bilinear matrix inequality;
IoT	Internet of things;
IQCs	Integral quadratic constraints;
LKF	Lyapunov-Krasovskii functional;
LMI	Linear matrix inequality;
LPV	Linear parameter varying;
LTI	Linear time-invariant;
MC	Monotonic convergent;
MPC	Model predictive control;
NCS	Networked Control System;
non-MC	Non-monotonic convergent;
PDC	Parallel distributed compensation;
PLC	Programmable logic controller;
PMSL	Position-type feedback model with speed limitation;
SOS	Sum-of-squares;
T-S	Takagi-Sugeno;
ZOH	Zero-order-holder;
s	Seconds;
cm	Centimeters;

List of Notations

$\mathbb{R}$	set of real numbers;
$\mathbb{R}^n$	the set of vectors of dimensions n with real entries;
$\mathbb{R}_+$	represents the set of non-negative real numbers;
$\mathbb{R}^{n \times m}$	the set of matrices of dimensions $n \times m$ with real entries;
$\mathbb{Z}$	set of integer numbers;
$\mathbb{N}$	set of natural numbers;
$\mathbb{N}^*$	set of natural number except zero;
$\mathbb{K}$	set of differentiable functions on an interval $[0, T]$ to $\mathbb{R}^n$, where $T \in \mathbb{R}_+$;
$M \in \mathbb{R}^{n \times m}$	a matrix with dimension $n \times m$ with real entries;
$M \in \mathbb{S}^n$	a symmetric matrix with dimension $n \times n$ with real entries;
$x \in \mathbb{R}^n$	a vector with n positions and real entries;
$M^\top$	the transpose of matrix M ;
$M_{(i)}$	the i -th row of matrix M ;
M_{ii}	the diagonal element (i, i) of a square matrix M ;
$M_{(i)}^\top$	stands for $(M_{(i)})^\top$;
$M > \mathbf{0}$	indicates that the square matrix M is symmetric positive definite;
$M \geq \mathbf{0}$	indicates that the square matrix M is symmetric positive semi-definite;
$\text{He}\{M\}$	$M + M^\top$
$\ \cdot\ $	the Euclidean norm of a vector or matrix;
$\ \cdot\ _2$	2-norm of a vector or matrix;
$\star$	represents the symmetric transposed blocks in square matrices;
$\mathbf{I}$	the identity matrix of appropriate dimensions;
$\mathbf{0}$	the null matrix of appropriate dimensions;
$\mathcal{E}(P, \zeta)$	an ellipsoidal set is defined as $x^\top(k)Px(k) \leq \zeta^{-1}$ for a given $P > 0$;
$A \subset B$	A is a subset of B ;
$A \subseteq B$	A is a subset of or equal to B ;
$\text{diag}\{A, B\}$	denotes is a block diagonal matrix of A and B matrices;
$\text{Co}\{\cdot\}$	denotes the convex hull of a set;
$\mathbb{I}_d$	denotes the set of nonnegative integers from 0 to d , $\forall d \in \mathbb{N}$;
$\mathbb{I}_d^*$	denotes the set of positive integers from 1 to d , $\forall d \in \mathbb{N}^*$;
Δf	$\bar{f} - \underline{f}$, with $\bar{f} \geq \underline{f}$ for any real scalar function $f : \mathbb{R} \rightarrow [\underline{f}, \bar{f}]$;
$\lfloor \gamma \rfloor$	floor (nearest integer towards minus infinity) of any real number $\gamma \in \mathbb{R}$;
$\lceil \gamma \rceil$	ceiling (nearest integer towards infinity) of any real number $\gamma \in \mathbb{R}$;
$\mathbf{1}_{\mathbb{N}}$	denotes the indicator function of the subset $\mathbb{N}$ of $\mathbb{R}$, i.e., $\mathbf{1}_{\mathbb{N}}(d) = 1$ if $d \in \mathbb{N}$, 0 otherwise;
C_k^n	denotes the binomial coefficient of two non-negative integers $n \geq k \geq 0$, $\binom{n}{k} = \frac{n!}{k!(n-k)!};$
$\otimes$	denotes the Kronecker product;
$f \in \mathcal{C}^1$	denotes that the function f is continuously differentiable.

General Introduction

Everyone must leave something behind when he dies, my grandfather said. A child or a book or a painting or a house or a wall built or a pair of shoes made. Or a garden planted. Something your hand touched some way so your soul has somewhere to go when you die, and when people look at that tree or that flower you planted, you're there.

It doesn't matter what you do, he said, so long as you change something from the way it was before you touched it into something that's like you after you take your hands away. The difference between the man who just cuts lawns and a real gardener is in the touching, he said. The lawn-cutter might just as well not have been there at all; the gardener will be there a lifetime.

Fahrenheit 451 (1953)
RAY BRADBURY

This doctoral research was conducted under a joint co-supervision agreement between the Centro Federal de Educação Tecnológica de Minas Gerais (CEFET-MG, Brazil) and the Université de Reims Champagne-Ardenne (URCA, France).

The recent increase in autonomous systems, both in industry and in everyday tasks of society, creates a new way to interact with and utilize digital devices. Whether in the scope of industry, services, entertainment, or transportation, system automation allows for the execution of automatic tasks and real-time monitoring of sensors and actuators. From such trends, new concepts arose in the last decade relating to the integration and interaction of devices connected through communication networks (Liu et al., 2019). Regarding industry and manufacturing, the concept of Industry 4.0 includes cyber-physics autonomous systems connected through networks with access to a data cloud that allows faster and more intelligent solutions with reduced costs. Similarly, autonomous and connected devices are also more present in the quotidian lives of human beings through the Internet of Things (IoT) in the context of residential automation, for instance. However, the system remains connected to a network usually limited in speed and bandwidth, which can cause network congestion, high energy consumption, and data packet loss. In this context, the efficient use of network resources is a requirement in control systems. Figure 1.1 illustrates the typical architecture of a Networked Control System (NCS). The physical continuous-time process (plant) is monitored by sensors and manipulated by actuators. The information collected by the sensors is transmitted to the discrete-time digital controller via a

communication network, which can be implemented using either wired or wireless technologies. Similarly, the control signals computed by the controller are sent back through the network to the actuators, closing the control loop. This configuration provides greater flexibility, integration, and scalability in automation systems, but also introduces challenges related to communication delays, packet losses, and bandwidth limitations.

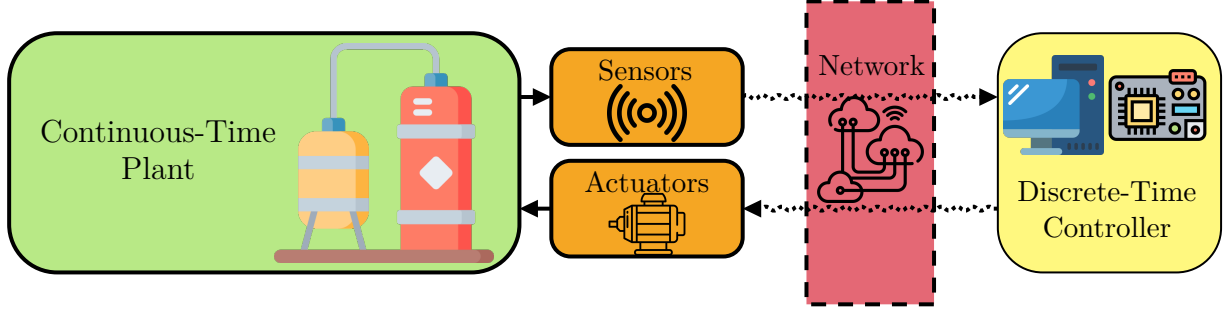


Figure 1.1: Typical architecture of a networked control system. The continuous-time process is monitored by sensors and driven by actuators, which interact with the discrete-time controller through a shared communication network. The network can be either cabled or wireless, enabling data exchange among distributed components.

In general, the dynamics of controlled devices are described by continuous-time differential equations. With the wide availability of processors, controllers are commonly implemented on digital platforms. From the literature, different approaches are widely employed to design digital controllers for continuous-time plants, which can be classified according to the characterization of the sampling interval, the adopted design strategy, and the associated stability guarantees (Nešić et al., 1999).

The first approach is based on a continuous-time plant model, followed by the discretization of the controller, which is often a practical choice in engineering applications. The second approach involves exact or approximate discretization of the continuous-time plant model, e.g., via the Euler method, while generally disregarding inter-sample behavior. A discrete-time controller is then designed based on the resulting discretized model. Although standard discretization techniques are widely used, they provide approximations that may fail to capture inter-sampling dynamics, potentially compromising stability, particularly for large or aperiodic sampling intervals. For linear systems, exact discrete-time models can often be derived, making this approach straightforward. For nonlinear systems, however, exact discrete-time models are typically unavailable or difficult to obtain, so approximate discretization methods are more commonly employed. A central question in both approaches is whether the designed controllers guarantee stability of the original continuous-time plant and, if so, in what sense. Figure 1.2 illustrates a networked control system for a nonlinear plant designed using a discrete-time model-based approach. The continuous-time plant model explicitly captures the nonlinear dynamics of the system. Transitioning to a discrete-time representation introduces certain approximations: the discrete-time plant model neglects inter-sampling dynamics, which may result in model errors if the sampling rate is insufficiently high. Furthermore, the discrete-time controller is inherently vulnerable to

aperiodic sampling caused by packet loss or network-induced transmission delays. These factors must be carefully considered during the design and implementation of digital controllers to ensure that stability and performance are preserved in practical networked environments.

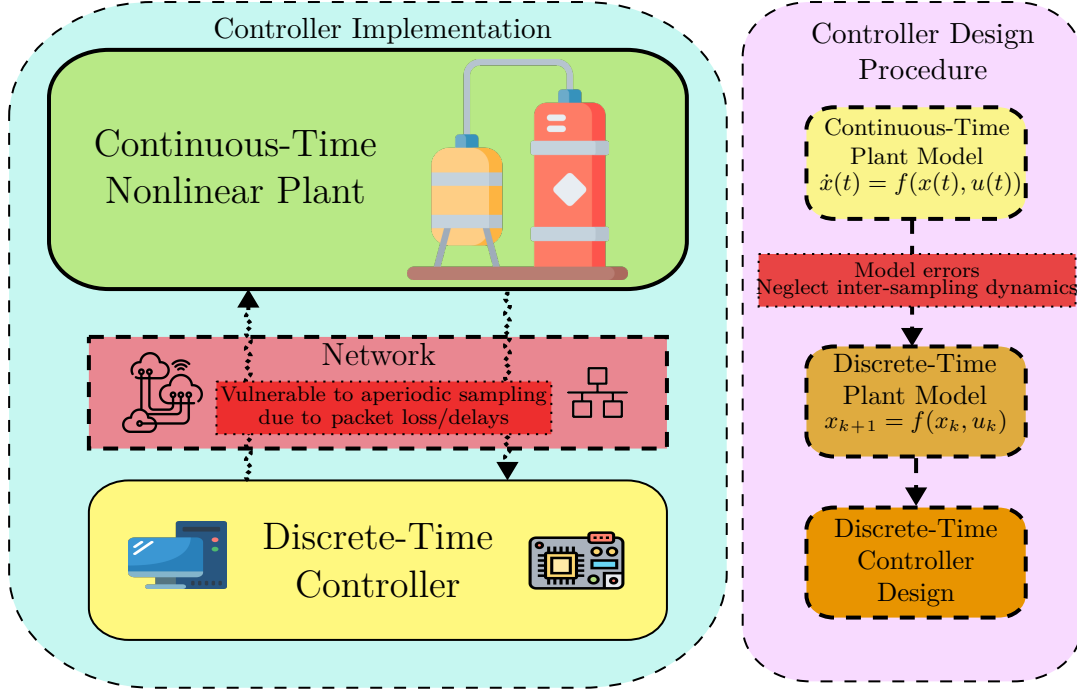


Figure 1.2: Schematic overview of the control and design chain for a continuous-time networked nonlinear plant, highlighting key limitations of discrete-time direct design implementations, showing typical model errors due to discretization and vulnerabilities introduced by aperiodic sampling, packet loss, or communication delays.

To illustrate how discrete-time models of nonlinear systems may fail to faithfully represent the real system, particularly when larger sampling intervals are required, consider the coupled tank system shown in Figure 1.3. Similar systems can be found in chemical processes, the food industry, pharmaceuticals, petrochemicals, thermal power plants, and others. In all these industries, level control is a common requirement, often exhibiting nonlinear dynamics. Therefore, the coupled-tank process presented in Figure 1.3 poses a control challenge due to the nonlinearity introduced in Tank T_3 , as shown in Figure 1.3(b). In particular, consider the configuration with two coupled tanks depicted in Figure 1.3(c), whose nonlinear dynamics can represent stages or processes in the aforementioned industries.

The system is modeled¹ based on mass balance equations and Bernoulli's principles as follows:

$$\begin{aligned} \dot{h}_1(t) &= \frac{q_{in}(t)}{A_1(h_1(t))} - \frac{h_1(t) - h_2(t)}{R_{12}(h_1(t), h_2(t))A_1(h_1(t))}, \\ \dot{h}_2(t) &= \frac{h_1(t) - h_2(t)}{R_{12}(h_1(t), h_2(t))A_2} - \frac{q_{out}(t)}{A_2}, \end{aligned} \quad (1.1)$$

where $h_1(t)$ and $h_2(t)$ are the water levels of the respective tanks, the flow equations were

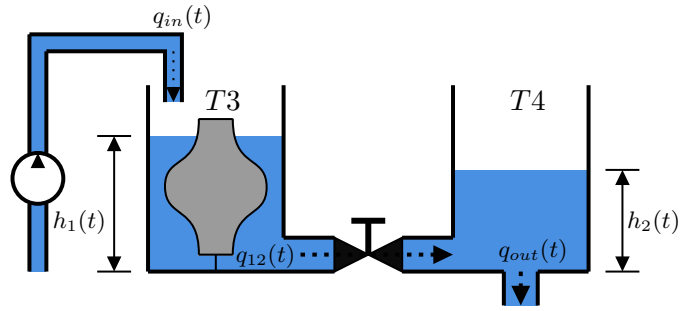
¹It is worth noting that, during the course of the research, the system underwent maintenance interventions, which led to changes in its parameters. For this reason, the equations are reintroduced in each occurrence, reflecting the updated dynamics.



(a) Coupled tank system.



(b) Solid contained in tank T3



(c) Tank system configuration.

Figure 1.3: Coupled tank system with nonlinear dynamics.

experimentally obtained as:

$$q_{in}(t) = K_b u(t) \text{ cm}^3/\text{s},$$

where the input flow is controlled by the signal $u(t)$. Experimental tests allow to identify $K_b = 26.8934$ and

$$q_{out}(t) = 0.0041h_2(t)^3 - 0.4451h_2(t)^2 + 36.4733h_2(t) + 507.1889 \text{ cm}^3/\text{s},$$

representing the output flow. The flow between the tanks depends on the hydraulic resistance given by

$$R_{12}(h_1(t), h_2(t)) = 0.1192(h_1(t) - h_2(t))^3 - 2.45(h_1(t) - h_2(t))^2 + 60.2987(h_1(t) - h_2(t)) + 449.7407 \text{ s/cm}^2.$$

The cross-sectional area of tank T4 is given by $A_2 = 3019 \text{ cm}^2$, whereas the area of tank T3 is described by

$$A_1(h_1(t)) = \frac{3r}{5} \left(2.7r - \frac{\cos(2.5\pi((h_1(t) - 8) \times 10^{-2} - \psi))}{\omega\sqrt{2\pi}} \times e^{-\frac{((h_1(t) - 8) \times 10^{-2} - \psi)^2}{2\omega^2}} \right) \times 10^4 \text{ cm}^2,$$

with $r = 0.31 \text{ m}$, $\omega = 0.55$, and $\psi = 0.4$. For more details on the modeling of this system, see [Quadros et al. \(2020\)](#); [Lopes et al. \(2020b\)](#); [Oliveira et al. \(2022\)](#).

The model (1.1) has been discretized using the forward Euler method with two different sampling times: $T_s = 4 \text{ s}$ and $T_s = 30 \text{ s}$. Figure 1.4 shows the tank levels obtained from a real-time open-loop experiment alongside the discrete-time model responses for two step changes in the input signal. The curve corresponding to $T_s = 4 \text{ s}$ closely follows the experimental data, whereas

the model with $T_s = 30$ s deviates significantly from the actual system behavior. This difference highlights a fundamental challenge in nonlinear system modeling and control: how to accurately discretize a model with inherent nonlinear dynamics. A key aspect lies in the assumption of regular sampling, i.e., obtaining measurements at uniform intervals. The emergence of Industry 4.0 introduces a new paradigm, integrating digital communication networks between the plant and the controller. In this context, even dedicated systems may experience packet losses, delays, or other network-induced irregularities, breaking down with the fundamental hypothesis of regular sampling behind any conventional digital controller approach. A pertinent example is the dedicated data networks in modern aircraft, where packet losses can occur despite the high reliability requirements of the system. Such considerations reinforce the need for control strategies that explicitly account for inter-sampling dynamics and network imperfections.

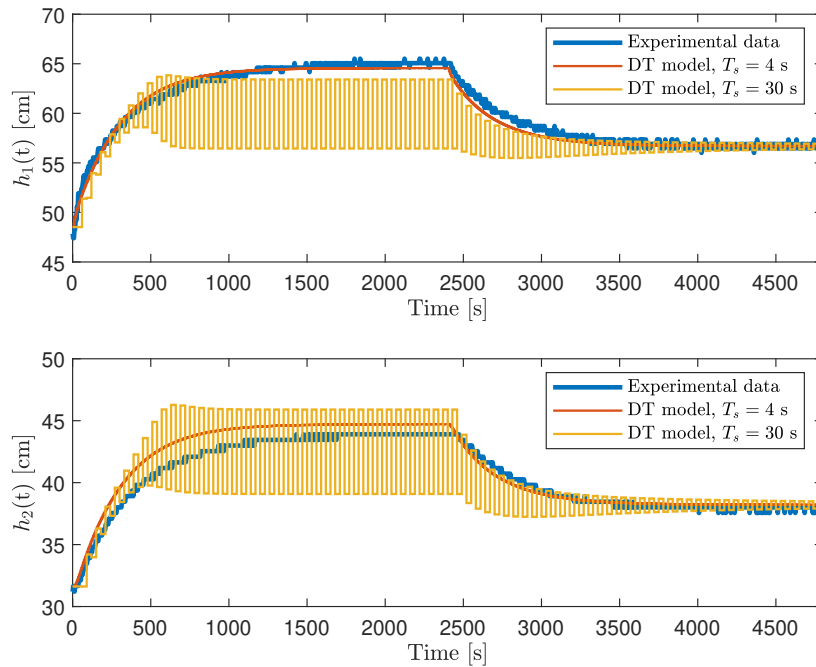


Figure 1.4: Comparison between the discrete-time model and the experimental data of the tank system with nonlinear dynamics.

An alternative approach is based on the sampled-data model of the plant, which explicitly accounts for inter-sampling dynamics in the controller design. This method overcomes the limitations of direct discretization by ensuring that the continuous-time behavior between samples is preserved. In practical implementations, sampled-data models allow aperiodic sampling and exhibit robustness to data packet loss, transmission delays, and network congestion, further reinforcing their suitability for networked control systems (Liu et al., 2019). In this context, efficient control strategies are required to address the challenges of real-time operation. Thus, sampled-data control emerges as a promising solution, explicitly modeling the continuous-time plant dynamics while applying the control signal in discrete-time. Furthermore, the use of larger sampling intervals reduces communication load on the network and saves energy, which is partic-

ularly beneficial in resource-constrained or networked control applications. Figure 1.5 presents the schematic overview of the sampled-data control system for a continuous-time nonlinear plant.

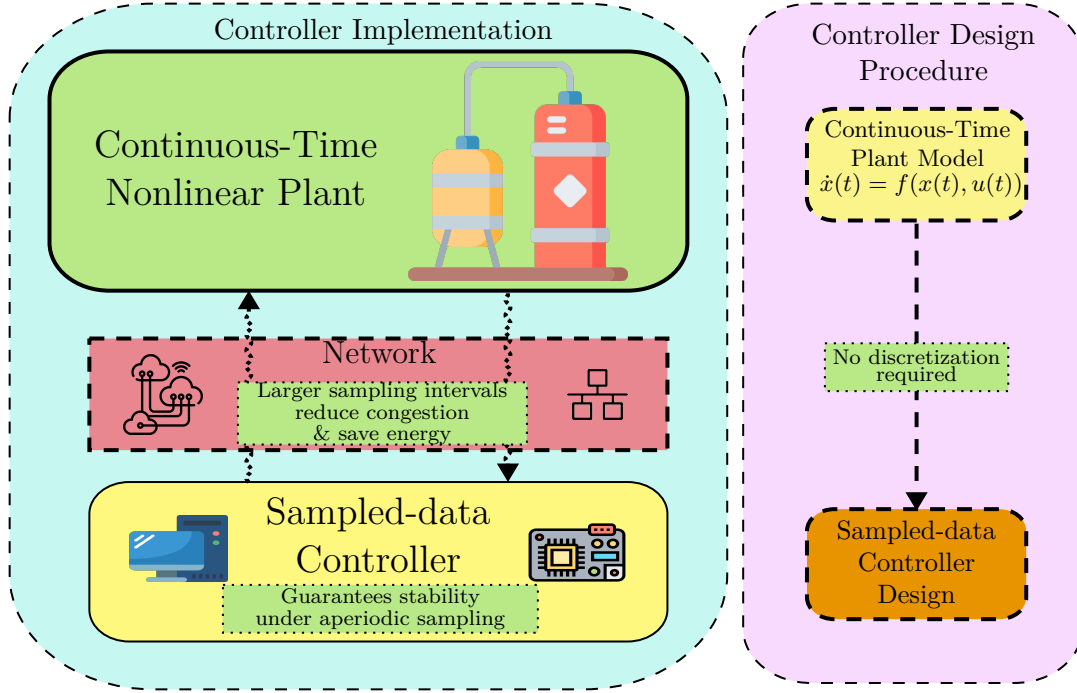


Figure 1.5: Overview of the sampled-data control implementation for a continuous-time nonlinear plant. Key features of the controller include stability guarantees under aperiodic sampling, no need for discretization of the plant model, and the possibility of using larger sampling intervals, which reduces network congestion and saves energy.

As mentioned, real control systems can have nonlinear dynamics, which makes it challenging to design controllers for systems of this type. Therefore, time-varying parameters modeling is a widely employed technique for representing uncertain or nonlinear systems (Karimi et al., 2007). Such an approach employs a set of local linear systems weighted by parameters or membership functions, enabling the consideration of nonlinearities or uncertainties while maintaining a relatively simple mathematical representation. Such a class of systems can be described as polytopic models, including Takagi-Sugeno (T-S) fuzzy systems (Takagi and Sugeno, 1985) and linear parameter varying (LPV) or *quasi*-LPV systems (Shamma and Cloutier, 1993). LPV models are commonly used to handle uncertainties or changes in system dynamics that depend on time-varying parameters, which are often supposed to be available online. On the other hand, T-S fuzzy and *quasi*-LPV models provide an exact representation of the nonlinearities present in the system through the sector nonlinearity approach. In fact, T-S fuzzy and *quasi*-LPV systems can be viewed as a particular case of LPV systems when the time-varying parameters depend on the states of the system (Briat, 2015, pp. 5). The key benefit is to be able to deal with this class of systems with the same methods that are used in linear systems. Therefore, several problems of stability analysis (at least in a local sense), controller or observer design, can be formulated as convex optimization problems, grounded in Lyapunov stability theory (Khalil, 2002) and expressed through linear matrix inequalities (LMIs) (Boyd et al., 1994). Note that

several approaches of stability analysis and control design for nonlinear systems can be formulated as convex optimization problems, based on Lyapunov stability theory, with some degree of conservatism. The critical issue in both cases is the availability of information concerning the state and the parameter, which allows for a more elaborate controller design and can provide high performance indices in the closed-loop system. Therefore, the use of polytopic systems provides a broad framework for representing the dynamics of a wide range of systems, from medical applications to industrial engineering implementations, including cyber-physical systems, new event-triggering techniques, and systems under constraints requiring high-fidelity models (Oliveira et al., 2017; Tasoujian et al., 2020; Colmegna et al., 2021; Vafamand et al., 2019; Pessim and Lacerda, 2022; de Souza et al., 2022; Lacerda, 2022).

Sampled-data control in systems with time-varying parameters is a challenging task. As previously mentioned, the states of the plant evolve continuously in time, while the controller operates in the discrete-time domain. Although the synthesis of time-varying controllers can yield more effective and robust results than static ones, in sampled-data systems, the scheduling parameters, measured by the plant sensors and responsible for governing the system dynamics, also evolve continuously but must be sampled for use by the controller. This leads to a mismatch or asynchronous behavior between the plant parameters and those used by the controller at sampling instants, which can result in overly conservative solutions (Gomes da Silva Jr. et al., 2015; Palmeira et al., 2022). Another critical aspect in reducing conservatism in the design of sampled-data controllers is the selection of a Lyapunov-Krasovskii functional (LKF) within the time-delay framework (Fridman, 2010; Seuret, 2012). Typically, LKFs are proposed to incorporate more information about the system states and the delay. To guarantee stability, the derivative of these functionals must be negative definite. In cases where the derivative is a linear function of the time-varying delay, negativity is ensured by the convexity of the delay interval. However, when an LKF is chosen such that its derivative is a second-degree (quadratic) polynomial with respect to the time-varying delay, ensuring negative definiteness is nontrivial, as the coefficients of such functions are matrices composed of decision variables that must guarantee the negativity of the resulting polynomial over the specified delay interval. Consequently, criteria based on geometric interpretations, used to define the negativity of quadratic functions over a time-varying delay interval, have been employed to provide less conservative stability and stabilization conditions for time-delay and sampled-data systems using LMIs (Zhang et al., 2022b; Xie et al., 2023; He et al., 2023; Liu et al., 2023).

Despite the ability of polytopic models to capture the dynamics of nonlinear systems, certain phenomena require specialized approaches for accurate analysis. This work focuses on rate (or slew-rate) and magnitude constraints due to actuators, which are common in real control systems due to physical, safety, or power limitations. If not properly addressed, these limitations can degrade performance, generate spurious equilibrium points, or even induce unstable behavior in closed-loop systems (Tarbouriech et al., 2011, pp. 5). Notable incidents attributed to rate-limiting saturation effects include air crashes (Klyde et al., 1997; Duda, 1997) and the infamous Chernobyl power plant explosion (Stein, 2003). Moreover, slew-rate saturation is a significant

concern in power electronics, as it may constrain the performance of such devices (Gacio et al., 2012). However, despite the recognized importance of accounting for rate-of-change saturation, it has not been addressed in the literature to the same extent as magnitude saturation, leaving room for further research exploration, including the sampled-data control of LPV systems.

This thesis seeks to investigate several aspects mentioned above, individually, for specific objectives and jointly, aiming to propose new sampled-data control design methods for LPV systems with actuators under saturation in the magnitude and rate of variation. First, a controller design of a continuous-time system based on the discrete-time model is proposed, considering saturating actuators, thereby utilizing the more conventional framework of regular sampling. Even with the limitations of this method regarding the sampling intervals, new design conditions are proposed with real-time experiments. Next, this thesis is dedicated to the sampled-data control framework, which ends with the sampled-data control of LPV systems under magnitude and rate saturating actuators. Therefore, a direct alternative to the propositions of the first conventional approach.

Next, the general and specific objectives of this work are listed.

1.1 Objectives

The main objective of this thesis is to develop new LMI-based conditions for synthesizing novel parameter-dependent state feedback sampled-data controllers that guarantee the asymptotic stability of LPV systems with actuators subject to saturation in magnitude and rate of change.

Some specific goals are also enumerated to complement the main objective of this thesis, as follows:

- I. To propose a parameter-dependent state-feedback controller design method for discrete-time LPV systems subject to actuator rate and magnitude saturation and magnitude-bounded disturbances;
- II. To propose new negative definiteness conditions to handle the negative definiteness of quadratic polynomials, aiming at less conservative analysis and synthesis conditions in sampled-data control in LPV systems;
- III. To develop new methods to address the mismatch between the time-varying parameters of the continuous-time plant and the discrete-time controller in sampled-data LPV systems, and explore this method jointly with the negative definiteness of quadratic polynomials;
- IV. To develop sampled-data control design conditions for LPV systems with actuator rate and magnitude saturation, incorporating the results of II and III.

1.2 Thesis Outline and Contributions

The subsequent chapters are organized in chronological order, reflecting the natural pro-

gression of the research activities, ideas, methods, and results throughout the doctoral period. Early chapters introduce fundamental concepts and review relevant literature, while subsequent chapters present methodological developments, theoretical results, and practical applications. This organization allows the reader to follow the evolution of the research and understand how each stage contributed to achieving the main objectives of the thesis.

This thesis is organized into six chapters. The current chapter provides the context and motivation for the research, as well as outlines the main and specific objectives. Chapter 2 presents a basic introduction and a literature review of the key topics related to this work.

Chapter 3 presents the first contribution of this thesis, which involves developing novel convex parameter-dependent state-feedback controller synthesis conditions that ensure regional and input-to-state stability, thereby effectively mitigating the adverse effects of rate- and magnitude-saturating actuators. The ℓ_2/ℓ_∞ gain between the disturbance input and the controlled output is considered, offering performance specifications and maintaining validity for specific initial conditions and amplitude-limited input disturbances. A position-type feedback model with speed limitation and the generalized sector condition is employed to handle saturation effects and derive the proposed controller synthesis method. Numerical examples are provided for comparative evaluation with existing literature across a broad spectrum of systems, including LPV, *quasi*-LPV, and T-S fuzzy systems. Additionally, a real-time experiment conducted on a nonlinear coupled tank system demonstrates the efficacy and feasibility of the proposed method for real-world implementation. The results of this chapter are published in Oliveira et al. (2024) and address Objective I.

Chapter 4 presents the second main contribution of this thesis, which addresses the problem of establishing negative definiteness conditions for quadratic functions, a common challenge in control problems involving time-varying delays. While existing geometric approaches provide relaxations based on partitioning techniques, they may lack monotonic convergence, casting doubt on their optimality. This chapter proposes novel and less conservative methods that ensure recursive monotonic convergence, in particular for LPV systems. The proposed conditions are derived by rewriting polynomial inequalities in homogeneous form and applying either Young's inequality or Pólya's theorem. Additionally, a Bézier representation and the de Casteljau algorithm are used to introduce further relaxed and monotonic convergent conditions. The effectiveness of the proposed approaches is demonstrated in the stability analysis of sampled-data control in LPV systems, with numerical examples validating their improvements over existing methods. These results contribute directly to Objective II and are based on the developments presented in Oliveira et al. (2025a).

Chapter 5 presents the third contribution of this thesis, which proposes a new approach to address the parameter mismatch problem inherent in the design of parameter-dependent sampled-data controllers for LPV systems. By leveraging the input-delay approach, the proposed method incorporates the mean value theorem while explicitly incorporating the delay into the stability conditions, resulting in less conservative synthesis conditions. The method enables larger admissible sampling intervals while maintaining stability guarantees. Its effectiveness and

superiority over existing solutions are demonstrated through a representative numerical example. The result is based on the published paper [Oliveira et al. \(2025b\)](#). Furthermore, this method is explored jointly with the negative definiteness of quadratic polynomials in the sampled-data controller synthesis case. The proposed design conditions are adapted to handle *quasi*-LPV systems and applied to a nonlinear level control system. Experimental tests on a nonlinear system illustrate the performance of the designed controller for different time-varying sampling intervals. The results of this chapter fulfill the requirements of Objective III.

Chapter 6 presents the fourth main contribution of this thesis, which concerns the synthesis of parameter-dependent aperiodic sampled-data state-feedback controllers for LPV systems with actuators subject to magnitude and rate saturation. Different from the results presented in Chapter 3, there is no need to obtain a discrete-time model of the plant, and the controller is resilient to aperiodic sampling intervals. The proposed method combines the looped-functional framework with a parameter-dependent generalized sector condition. Local stability is ensured through a negative definiteness condition of quadratic polynomials. Numerical examples are provided to compare the proposed conditions with existing results. The results in this chapter are aligned with Objectives IV and published in [Oliveira et al. \(2025c\)](#). An experimental validation on a nonlinear system with actuator limitations is performed, demonstrating the application of the proposed method.

Chapter 7 provides a brief overview of other works conducted during the doctoral research that are related to this thesis. The chapter also presents the concluding remarks and outlines perspectives for future research.

Fundamental Concepts

C'est l'histoire d'un homme qui tombe d'un immeuble de 50 étages.
 Au fur et à mesure de sa chute, il se répète sans cesse pour se rassurer :
 « Jusqu'ici tout va bien. Jusqu'ici tout va bien. Jusqu'ici tout va bien. »
 Mais l'important, ce n'est pas la chute, c'est l'atterrissage.

La Haine (1995)
 dir. MATHIEU KASSOVITZ

The purpose of this chapter is to outline essential theoretical concepts that support and clarify the main themes explored in this thesis. To achieve this, an overview is provided on the control of parameter-dependent systems, the use of Lyapunov theory for stability analysis, the use of the input-delay approach to model sampled-data systems, and specific techniques to handle actuator saturation.

2.1 State Feedback Control of Parameter-Dependent Systems

The stability analysis and controller synthesis for parameter-dependent systems have been extensively investigated in control systems (Rotondo et al., 2015). The main reason is that these models can accurately represent the dynamics of a broad class of processes, where parameters may change over time (or with the operating conditions). Moreover, parameter-dependent models can capture nonlinear dynamics while maintaining a simple mathematical representation using a convex set of linear models (Rugh and Shamma, 2000). Among this type of system, two approaches stand out in the design of parameter-dependent controllers, incorporating the system dynamics in the stabilizing gains. These are the linear parameters varying (LPV) systems (Briat, 2015) and the Takagi-Sugeno (T-S) Fuzzy systems (Takagi and Sugeno, 1985). These kinds of gain-scheduled control differ from the robust control framework where the uncertain parameters are not available in real-time (Oliveira et al., 2011).

The first reference to LPV systems was made by Shamma and Athans (1990), who investigated the analysis and synthesis of gain-scheduled controllers. Shamma and Athans (1992) subsequently addressed the challenges of applying gain-scheduled controllers in LPV systems by restricting slow parameter variations to ensure closed-loop stability. Consequently, numerous

studies on this category of systems have emerged, as outlined in surveys by (Leith and Leithead, 2000; Rugh and Shamma, 2000), providing a comprehensive overview of the state-of-the-art in gain-scheduling control in 2000. When the varying parameters are dependent on endogenous signals, the systems are called *quasi*-LPV, unlike pure LPV systems, where the parameters are dependent on exogenous signals (Sename et al., 2013, pp. 5). This type of system covers extensive practical applications as autonomous vehicles (Hachem et al., 2023), long-range guided projectiles (Venco et al., 2022), automotive suspensions (Basargan et al., 2023), aircraft (Liu et al., 2024), blood pressure control (Tasoujian et al., 2020), manipulators (Li et al., 2023), among others (Hoffmann and Werner, 2015). Notably, the LPV framework is still one of the main tools used in control systems. For more details, see (Hoffmann and Werner, 2015; Briat, 2015; Sename et al., 2013; Sename, 2025).

Conversely, T-S fuzzy systems were introduced by Takagi and Sugeno (1985), representing nonlinear systems in a validity domain using fuzzy sets, rules, and linear subsystems. Instead of time-varying parameters, the local models are weighted with fuzzy membership functions. As in LPV systems, T-S fuzzy systems have also been used in a vast application field such as autonomous driving (Alcalá et al., 2022), wind energy (Gandhi and Joo, 2022), semi-active automotive suspensions (Fang et al., 2024), biomechanics (Blandeau et al., 2024), medical treatments (Kashkynbayev and Rakkiyappan, 2024), and more (Precup et al., 2024). As the *quasi*-LPV systems, the membership functions are dependent on the endogenous signals, as the system states. Nevertheless, as pointed out by Rotondo et al. (2015), both approaches share similarities, particularly when the LPV systems exhibit affine dependence on the scheduling parameters. However, it is worth noting that LPV models may also feature non-affine parameter dependencies. However, both fields were developed independently, and both theories are not always considered the same. As pointed out by (Li et al., 2021a), T-S fuzzy systems and polytopic *quasi*-LPV systems can be considered as the same thing, except for the historical backgrounds of each, the robust control theory in the LPV framework, and the fuzzy logic theory in the T-S one. Therefore, many of the results obtained in one framework can be applied in the other. See (Rotondo et al., 2015) to better understand the parallelism of both approaches. It is also worth mentioning that in *quasi*-LPV and T-S fuzzy systems conditions, it is essential to consider the domain of validity of the models (Tanaka and Wang, 2001; Silva et al., 2020).

Consider a continuous-time parameter-dependent system given by:

$$\begin{aligned}\dot{x}(t) &= A(\alpha(t))x(t) + B(\alpha(t))u(t), \\ y(t) &= C(\alpha(t))x(t)\end{aligned}\tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the control signal vector, $y(t) \in \mathbb{R}^{n_y}$ is the output, the matrices $A(\alpha(t)) \in \mathbb{R}^{n \times n}$, $B(\alpha(t)) \in \mathbb{R}^{n_u \times n}$, and $C(\alpha(t)) \in \mathbb{R}^{n \times n_y}$ depend linearly on the time-varying parameter vector $\alpha(t)$. The matrices belong to a polytopic domain given by the convex combination of N known vertices:

$$\begin{bmatrix} A(\alpha(t)) & B(\alpha(t)) & C(\alpha(t)) \end{bmatrix} = \sum_{i=1}^N \alpha_i(t) \begin{bmatrix} A_i & B_i & C_i \end{bmatrix},\tag{2.2}$$

where $\alpha(t) \in \Gamma_N$ is the time-varying vector of parameters which is assumed to be online available (Briat, 2015, pp. 10) that belong to the unit simplex:

$$\Gamma_N = \left\{ \alpha(t) \in \mathbb{R}^N : \sum_{i=1}^N \alpha_i(t) = 1, \alpha_i(t) \geq 0, i = 1, \dots, N \right\} \quad (2.3)$$

Note that the vector of parameters $\alpha(t)$ lies over a fixed polytope where A_i , B_i , and C_i are the vertices. In other words, a polytopic system is constructed by a collection of linear systems weighted by measurable or estimable parameters $\alpha_i(t)$.

Assuming that $\alpha(t)$ is online available, a stabilizing state-feedback control law with time-varying parameter-dependent gains can be defined as:

$$u(t) = K(\alpha(t))x(t), \quad (2.4)$$

where $K(\alpha(t)) \in \mathbb{R}^{n_u \times n}$ and

$$K(\alpha(t)) = \sum_{i=1}^N \alpha_i(t) K_i, \quad (2.5)$$

with $\alpha(t) \in \Gamma_N$. The challenge, in this case, is to find LMI convex conditions to design the controller matrix gains $K(\alpha(t))$ to guarantee the asymptotic stability of (2.1).

However, many control system applications involve digital processors and devices. A direct alternative is to discretize model (2.1) with, for instance, Tustin discretization (Ogata, 1995) or forward Euler method (Chen, 1999). Based on the latter, the following result is obtained:

$$\dot{x}(t) = \lim_{T \rightarrow 0^+} \frac{x(t+T) - x(t)}{T}. \quad (2.6)$$

where T is the sampling period. Hence, with a small enough T , from (2.1) it follows that:

$$\begin{aligned} x(t+T) &\approx (\mathbf{I} + TA(\alpha(t_k)))x(t_k) + TB(\alpha(t_k))u(t_k), \\ y(t_k) &= C(\alpha(t_k))x(t_k), \end{aligned} \quad (2.7)$$

where the discrete-time parameter-dependent model can be written as:

$$\begin{aligned} x(k+1) &= \mathcal{A}(\alpha(k))x(k) + \mathcal{B}(\alpha(k))u(k), \\ y(k) &= \mathcal{C}(\alpha(k))x(k), \end{aligned} \quad (2.8)$$

at each sampling instant $t_k = kT$, with $k \in \mathbb{N}$, and $\mathcal{A}(\alpha(k)) = \mathbf{I} + TA(\alpha(t_k))$, $\mathcal{B}(\alpha(k)) = TB(\alpha(t_k))$, and $\mathcal{C}(\alpha(k)) = C(\alpha(t_k))$ (with the same convex hull). As it happens in the continuous-time model, the matrices of the discrete-time system belong to a polytopic domain given by the convex combination of N known vertices:

$$\begin{bmatrix} \mathcal{A}(\alpha(k)) & \mathcal{B}(\alpha(k)) & \mathcal{C}(\alpha(k)) \end{bmatrix} = \sum_{i=1}^N \alpha_i(k) \begin{bmatrix} A_i & B_i & C_i \end{bmatrix}, \quad (2.9)$$

and the vector of time-varying parameters $\alpha(k)$ is supposed to be available or measured online, belonging to the unit simplex (2.3). In consequence, a state-feedback discrete-time control law can be defined as:

$$u(k) = K(\alpha(k))x(k), \quad (2.10)$$

where $K(\alpha(k)) \in \mathbb{R}^{n_u \times n}$ and

$$K(\alpha(k)) = \sum_{i=1}^N \alpha_i(k) K_i. \quad (2.11)$$

In Chapter 3, a design condition based on such a discrete-time model is proposed. However, even if such a strategy is still largely used (Moreira and Lacerda, 2022; Reis et al., 2022; Pandey et al., 2023; Moreira and Lacerda, 2022), the choice of the sampling period T may affect the accuracy of the representation of the continuous-time model, as also discussed in Chapter 1. See (Lopes, 2022, pp. 26) for examples of how the sampling period can affect the performance and stability of controllers based on the discrete-time model. Hence, sampled-data controllers

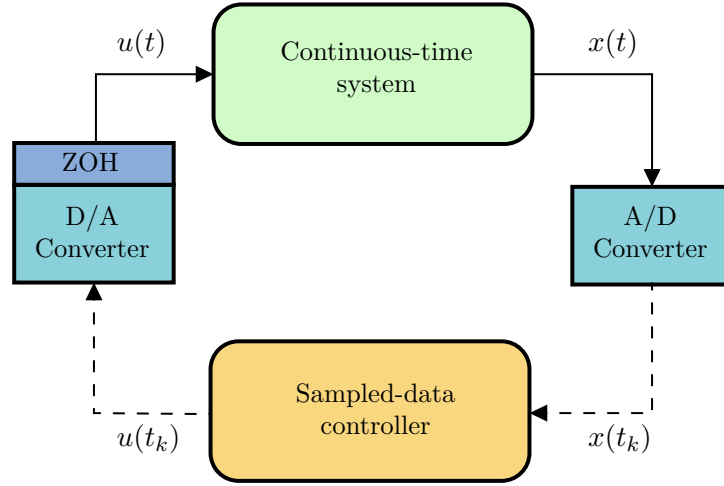


Figure 2.1: Schematic diagram of the sampled-data state feedback control structure: The plant states evolve in continuous time, $(x(t))$, while they are sampled aperiodically, $(x(t_k))$, by an analog/digital converter as well as the time-varying parameters, $(\alpha(t_k))$, measured by sensors. The controller is digitally implemented in a computer or microcontroller, and the input control signal, $(u(t_k))$ is sent to the plant after being converted into a digital signal and maintained until the next sampling instant by a zero-order-holder (ZOH).

arise as a suitable solution, providing a hybrid approach where a discrete-time controller can be obtained directly from the continuous-time model. Moreover, it is possible to ensure stability for aperiodic sampling instants in this approach, which is unsuitable for the direct discrete-time approach. A simple schematic of the sampled-data state feedback structure is shown in Figure 2.1.

Consider the continuous-time system (2.1), and a controller given by:

$$u(t) = K(\alpha(t_k))x(t_k), \quad (2.12)$$

for $t \in [t_k, t_{k+1})$ being t_k the sampling instant, where $K(\alpha(t_k)) \in \mathbb{R}^{n_u \times n}$ and:

$$K(\alpha(t_k)) = \sum_{i=1}^N \alpha_i(t_k) K_i, \quad (2.13)$$

with $\alpha(t_k) \in \Gamma_N$. The resulting state feedback closed-loop is then:

$$\begin{aligned} \dot{x}(t) &= A(\alpha(t))x(t) + B(\alpha(t))K(\alpha(t_k))x(t_k), \\ y(t) &= C(\alpha(t))x(t) \end{aligned} \quad (2.14)$$

Note that a mismatch exists between the inputs of the system and the gain matrices due to the parameter dependency. As mentioned by [Gomes da Silva Jr. et al. \(2015\)](#), even with $\alpha(t)$ and $\alpha(t_k)$ lying in the same polytopic structure, to consider that the parameters do not evolve during the sampling instants ([Tan et al., 2002](#)) can result in instability in real applications, since this assumption is not realistic. Conversely, assuming independence between the parameters, as done in [Shi and Su \(2014\)](#), is a very conservative solution. In order to find less conservative solutions, an approach was originally proposed in the LPV framework by [Gomes da Silva Jr. et al. \(2015\)](#) where the continuous-time parameter is divided into two parts: a sampled one, lying in the limits of the parameter, and a continuous-time one that is bounded by the intersample limits. It was extended in the context of magnitude input saturation ([Palmeira et al., 2018](#)), disturbance attenuation ([Gomes da Silva Jr. et al., 2018](#)), *quasi*-LPV systems ([Palmeira et al., 2021](#); [Coutinho et al., 2021](#)), and model predictive control ([Palmeira et al., 2022](#)). In the framework of T-S fuzzy systems, [Lopes et al. \(2020a\)](#) proposed a new relaxation scheme based on Tuan’s lemma ([Tuan et al., 2001](#)) to reduce conservatism and extended it for input saturation cases ([Lopes et al., 2021](#)). The standard strategies to handle the mismatch in sampled-data control with parameter-dependent controller gains are discussed in Chapter 5, and a new approach is proposed to obtain less conservative results.

2.2 Lyapunov Stability

The Lyapunov stability theory is an essential tool in the stability investigations and controller designs for parameter-dependent systems, allowing convex formulations regarding LMIs. The advantage of using LMIs is the efficiency of numerical resolutions and the guarantee of optimal solutions under convex objective functions ([Boyd et al., 1994](#)). Since then, several works based on different Lyapunov functions have been published in the literature. In [Bernussou et al. \(1989\)](#), the robust stabilization problem for uncertain continuous-time linear systems using quadratic Lyapunov candidate functions is addressed. In [Geromel et al. \(1991\)](#), robust stability and state feedback control are proposed using quadratic Lyapunov functions for continuous and discrete-time systems. [Geromel et al. \(1992\)](#) proposed an $\mathcal{H}_\infty$ guaranteed cost control solution for uncertain and continuous-time linear systems using a quadratic structure. In such approaches, the gains of the controller are constant, i.e., the controller parameters are time-invariant. Therefore, the controller is called robust, ensuring the closed-loop stability of uncertain polytopic systems.

To obtain less conservative solutions, some authors have proposed the use of parameter-dependent Lyapunov functions ([Feron et al., 1996](#); [Rantzer and Johansson, 2000](#)). In [Daafouz and Bernussou \(2001\)](#), the authors propose the robust polyquadratic stability of discrete-time systems with arbitrary parameter variation inside a polytope, finding an affine parameter-dependent Lyapunov function. The stability analysis of piecewise linear systems is addressed in [Feng \(2002\)](#) for discrete-time and [Johansson \(2003\)](#) for continuous-time systems. Considering piecewise Lyapunov matrices ([Xie et al., 1997](#)), [Almeida et al. \(2001\)](#) propose an evolutionary

algorithm for robust stability and stabilization in uncertain time-varying linear systems in the continuous-time domain, and Leite and Peres (2004) address the robust stabilization of linear discrete-time systems with uncertain parameters. Other significant contributions to the literature in this context can be found in Montagner et al. (2006); Oliveira and Peres (2009); De Caigny et al. (2009) and the references therein.

In the context of T-S fuzzy systems, similar approaches for the Lyapunov function candidate have been proposed in several contexts, such as (Johansson et al., 1999; Feng and Harris, 2001; Tanaka et al., 2003; Guerra and Vermeiren, 2004; Tanaka et al., 2007; Mozelli et al., 2009; Guerra et al., 2012; Cherifi et al., 2019) and references therein. In particular, relaxation lemmas to provide a limited set of LMIs and avoid checking all the vertices combinations received much attention in this context (Kim and Lee, 2000; Tanaka and Wang, 2001; Tuan et al., 2001; Sala and Ariño, 2007).

2.2.1 Parameter-Dependent Lyapunov Candidate Function

The stability of a linear system concerning its equilibrium point can be characterized by its internal energy. If the energy of the equilibrium point tends to zero when the time approaches infinity, the equilibrium point is considered asymptotically stable. Throughout the text, when the expression that a system is stable is used, the equilibrium point of interest is stable.

Lyapunov's method uses a scalar function representing the amount of energy in the system from its origin. If this function $V(x(t))$ is always positive, radially unbounded, null only at the origin, and with strictly negative variations for $V(x(t)) \neq 0$ and null for $V(x(t)) = 0$, the system trajectories converge towards the origin, i.e., the system is asymptotically stable, and it is called a Lyapunov function (Khalil, 2002, pp. 114). The procedure for certifying the Lyapunov asymptotic stability of a linear continuous-time system is illustrated as follows.

Consider the continuous-time parameter-dependent system given by (2.1) and analyze the non-forced configuration, i.e., with $u(t) = \mathbf{0}$, for an initial condition $x(0) \neq 0$. The stability can be demonstrated with the following theorem:

Theorem 2.1 (Adapted from Khalil (2002)). *If there exists V , being this a Lyapunov function, and with κ_1 , κ_2 , and κ_3 positive scalars, if the conditions:*

$$\kappa_1 \|x\| \leq V(x) \leq \kappa_2 \|x\|, \quad (2.15)$$

and

$$\dot{V}(x(t)) \leq -\kappa_3 \|x\|, \quad (2.16)$$

are fulfilled for all $x \in \mathbb{R}^n$, the continuous-time system (2.1) is asymptotically stable.

For discrete-time linear systems, (2.16) is replaced by:

$$V(x(k+1)) - V(x(k)) \leq -\kappa_3 \|x\|. \quad (2.17)$$

Then, the discrete-time system (2.8) is asymptotically stable.

Therefore, choosing a structure for a candidate Lyapunov function that satisfies Theorem 2.1 is needed. In continuous-time systems, for instance, one of the most used candidate Lyapunov functions is the quadratic function defined as:

$$V(x(t)) = x^\top(t)Px(t), \quad (2.18)$$

where $P \in \mathbb{R}^{n \times n}$. To guarantee the positivity of $V(x(t))$, matrix P must be symmetric and positive definite ($\mathbf{0} < P = P^\top$), i.e., all eigenvalues of P must be greater than zero. For function $V(x(t))$ to be decreasing, the derivative $\dot{V}(x(t))$ must be negative along the trajectories of the continuous-time system.

However, the quadratic function can be quite restrictive for time-varying systems such as system (2.1). Since the conditions formulated from this function are defined only on the polytope vertices, the results using this type of function can be very conservative. So, an alternative is to use time-varying parameter-dependent functions, making the function more general and possibly decreasing conservatism. Assume a parameter-dependent Lyapunov candidate function quadratic on the system state and dependent in a polytopic way on the time-varying parameter described as:

$$V(x(t)) = x^\top(t)P(\alpha(t))x(t), \quad (2.19)$$

$$P(\alpha(t)) = \sum_{i=1}^N \alpha_i(t)P_i > \mathbf{0}. \quad (2.20)$$

Then, the following lemma for the stability analysis of a continuous-time system is provided:

Lemma 2.1 (Gahinet et al. (1996)). *Consider the continuous-time system given by (2.1) and analyze the non-forced configuration. If there exists a differentiable positive definite symmetric matrix $P(\alpha(t))$ satisfying:*

$$A^\top(\alpha(t))P(\alpha(t)) + P(\alpha(t))A(\alpha(t)) + \frac{d}{dt}P(\alpha(t)) < \mathbf{0}, \quad (2.21)$$

then, (2.19) is a Lyapunov function satisfying Theorem 2.1 and system (2.1) is asymptotically stable.

Note that $P(\alpha(t))$ must be differentiable, and (2.21) takes into account the time-derivative of the parameter. To handle this term, some simplifications can be made, and it has been a source of conservatism in LMI solutions, vastly investigated in (Jadbabaie et al., 1998; Tanaka et al., 2003; Geromel and Colaneri, 2006; Chesi et al., 2007; Mozelli et al., 2009; Guerra et al., 2012; Vieira et al., 2023).

For the discrete-time domain, consider a parameter-dependent Lyapunov candidate function as:

$$V(x(k)) = x^\top(k)P(\alpha(k))x(k), \quad (2.22)$$

$$P(\alpha(k)) = \sum_{i=1}^N \alpha_i(k)P_i > \mathbf{0}. \quad (2.23)$$

The following lemma guarantees the stability of a discrete-time system:

Lemma 2.2 (Daafouz and Bernussou (2001)). *Consider the discrete-time system given by (2.8) and analyze the autonomous configuration. If there exists a positive definite symmetric matrix $P(\alpha(k))$ satisfying*

$$\mathcal{A}^\top(\alpha(k))P(\alpha(k+1))\mathcal{A}(\alpha(k)) + P(\alpha(k)) < \mathbf{0}, \quad (2.24)$$

then, (2.22) is a Lyapunov function satisfying Theorem 2.1 and system (2.8) is asymptotically stable.

Chapter 3 presents convex condition, based on LMIs, for the controller synthesis. The conditions are obtained through parameter-dependent Lyapunov candidate functions in the discrete-time domain.

2.3 Input-Delay and Looped-Functionals

The class of time-delay systems has been investigated since the beginning of control theory (Zhang et al., 2022b). In such a class, the dynamics depend not only on the present state but also on the past states. The delay can happen due to the nature of the system itself in transferring energy or information, due to the structural components of the control system, such as sensors, actuators, and network protocols, or even due to the intentional introduction of delay in the control law (Gu et al., 2003). An example of a time-delay system is the classical and illustrative case of a person taking a shower with the aim of achieving a specific water temperature by opening and closing the taps of cold and hot water (Kolmanovskii and Myshkis, 1992). The interval time the water takes to flow from the tap (actuator) to the skin of the bather (sensor) is a delay. Based on experience with this system, it has been observed that it is necessary to wait until the temperature reaches a steady-state value before applying further changes to the controller. Otherwise, the desired temperature may not be achieved, potentially resulting in slight overheating, abrupt thermal fluctuations due to cold water input, or even sustained oscillatory behavior. Despite the simple example, it is possible to see how delay can cause performance degradation and instability in controlled systems. Examples of systems with delays can be seen in mechanics, biological and chemical processes, and population dynamics, among other applications (Hale and Lunel, 1993; Gu et al., 2003; Richard, 2003; Fridman, 2014).

In particular, this thesis focuses on the modeling of sampled-data control systems through the framework of delay systems (Hespanha et al., 2007; Briat, 2011; Fridman, 2010), which constitute a fundamental building block for networked control systems (NCSs). However, the network is prone to issues such as data congestion, packet loss, and transmission delays, which may critically affect the stability and performance of networked or embedded control systems. In this context, aperiodic sampled-data control offers a flexible alternative to strict periodic communication, enhancing the ability of the system to operate reliably under variable and potentially adverse network conditions. To model such behavior, the effect of aperiodic sampling in continuous-time plant models can be captured using a time-delay system representation, as proposed in Fridman et al. (2004). In this framework, the control signal is modeled as a time-varying delayed input

that reflects the aperiodic sampling behavior. It is important to note that, in the context of time-delay systems, Theorem 2.1 (Lyapunov) does not guarantee stability. In general, approaches to ensuring the stability of systems with time delays are based on two notable results known as Lyapunov-Razumikhin (Razumikhin, 1956; Niculescu et al., 1995) and Lyapunov-Krasovskii (Krasovskii, 1963; Fridman and Shaked, 2003). Both are generalizations of the Lyapunov direct method, presented in Theorem 2.1, to time-delay systems. In general, approaches that use Lyapunov-Krasovskii are less conservative and more present in the literature. On the other hand, the Lyapunov-Razumikhin functions may be more straightforward to use (Fridman, 2014, pp. 52). Indeed, in most cases, it can be observed that stabilization conditions for such systems are obtained using LKFs (Fridman and Shaked, 2003). The survey Liu et al. (2019) recall some recent advances in NCS using the time-delay approach.

In addition to the input-delay approach, other representations are also used to deal with sampled-data systems and NCS. Due to continuous and discrete signals, the representation through hybrid dynamic systems (Goebel et al., 2012; Briat, 2021) is a natural path for sampled-data systems. In Fagundes et al. (2024b), the hybrid representation is used to investigate the stability of Lur'e systems with sampled-data control. Another approach is based on impulsive systems (Fiacchini and Morarescu, 2016), a subclass of hybrid systems. Huff et al. (2022b) used this characterization to represent sampled-data systems and obtain stability and stabilization conditions for precisely known systems under saturating actuators using quadratic Lyapunov functions. Other notable approaches to handle sample-data systems include the use of uncertain discrete-time models (Hetel et al., 2011), integral quadratic constraints (IQCs) (Turner and Gomes da Silva Jr., 2023), and passivity-based methods (Fujioka, 2009).

The input-delay approach is used in this thesis. Consider then the system presented in (2.14):

$$\begin{aligned}\dot{x}(t) &= A(\alpha(t))x(t) + B(\alpha(t))K(\alpha(t_k))x(t_k), \\ y(t) &= C(\alpha(t))x(t).\end{aligned}\tag{2.25}$$

The control signal $u(t) = K(\alpha(t_k))x(t_k)$ is maintained constant during intersampling by a zero-order hold (ZOH) function with a sequence of sampling instants:

$$0 = t_0 < t_1 < \dots < t_k < t_{k+1} < \dots, \lim_{k \rightarrow \infty} t_k = \infty,\tag{2.26}$$

where the sampling interval can be either constant $t_{k+1} - t_k \equiv \mathcal{T}$ or variable with k -dependency, $t_{k+1} - t_k = \eta_k$ (Fridman, 2014, pp. 273). As mentioned before, in NCS, it is reasonable to consider time-varying sampling periods due to practical applications. Hence, the aperiodic sampling instants η_k are limited by known minimum and maximal allowable values, $\mathcal{T}_1 \leq \mathcal{T}_2$, i.e., $0 < \mathcal{T}_1 \leq \eta_k \leq \mathcal{T}_2$, for all $k \in \mathbb{N}$.

From the input-delay approach, the system (2.25)-(2.26) can be considered as a continuous-time system with a piecewise linear time-varying delay. Hence, consider the sawtooth functions:

$$\tau(t) = t - t_k, \quad \forall t \in [t_k, t_{k+1}), \quad \forall k \in \mathbb{N},\tag{2.27}$$

illustrated in Figure 2.2. Note that $\dot{\tau}(t) = 1$, $\forall t \in [t_k, t_{k+1})$, $\forall k \in \mathbb{N}$. Then, from (2.27), a sampled signal $s(t_k)$ maintained by a ZOH can be replaced as $s(t - \tau(t))$. The closed-loop

By adding more information about the states and delays in the LKF, less conservative stability conditions may be obtained at the expense of more complex LKF candidates. See, for instance, the surveys about the construction of LKFs (Zhang et al., 2022b; Chen et al., 2022b). In general, the approaches to obtain less conservative LKFs are augmented functionals in which more information is used, such as derivatives and integrals of the state vector, or the state delay in the non-integral or integral terms (Chen et al., 2017, 2018; Seuret and Gouaisbaut, 2015; Zhang et al., 2017); multiple integral terms in which such terms are added to the LKF (Gyurkovics and Takács, 2016; Kim, 2016; Gyurkovics et al., 2017); and delay-partitioning method where the delay is divided into subintervals (Feng et al., 2015; Gouaisbaut and Peaucelle, 2007; Han, 2008).

The integrative term bounding is another key approach to obtaining less conservative LMI conditions with LKFs. Taking the derivative of (2.29) yields:

$$\begin{aligned}\dot{V}(t, x_t) = & \dot{x}^\top(t)Px(t) + x^\top(t)P\dot{x}(t) + x^\top(t)Qx(t) \\ & - x^\top(t - \tau(t))Qx(t - \tau(t))(1 - \dot{\tau}(t)) + \eta_k \dot{x}^\top(t)R\dot{x}(t) - \int_{t-\eta_k}^t \dot{x}^\top(s)R\dot{x}(s)ds.\end{aligned}$$

Note that the last integral term can be rewritten (Zhang et al., 2022b) as:

$$-\int_{t-\eta_k}^t \dot{x}^\top(s)R\dot{x}(s)ds = -\int_{t-\eta_k}^{t-\tau(t)} \dot{x}^\top(s)R\dot{x}(s)ds - \int_{t-\tau(t)}^t \dot{x}^\top(s)R\dot{x}(s)ds. \quad (2.30)$$

In the literature, using integral inequalities to establish an upper limit for quadratic integral terms as in (2.30) is usual. Jensen's inequality has been commonly applied in this context since the 2000s. Briat (2011) demonstrates how other inequalities of this type are equivalent. Seuret and Gouaisbaut (2013) proposed new conditions using Wirtinger's inequality to obtain tighter limits in time-varying delay systems. In the same context, Seuret and Gouaisbaut (2015) proposed using Bessel-Legendre polynomials, generalizing previous results and obtaining less conservative results with more decision variables.

In the sampled-data framework with aperiodic samplings, Seuret (2012) introduced a new strategy to build LKFs called looped-functionals. For that, looped terms that are equal at each sampling instant are considered, allowing the inclusion of non-positive terms in the LKF, aiming to obtain less conservative stability analysis conditions. The following theorem presents the stability criteria for sampled-data systems based on looped-functionals.

Theorem 2.3 (Seuret (2012)). *Consider the precisely known case of the sampled-data closed-loop system (2.28), i.e., there is no dependence on the time-varying parameter. Let $V : \mathbb{R}^n \rightarrow \mathbb{R}_+$ be a differentiable function satisfying*

$$\kappa_1 \|x\|^p \leq V(x) \leq \kappa_2 \|x\|^p, \quad (2.31)$$

being $p, \kappa_1 < \kappa_2$ positive scalars, and a continuous and differentiable functional $\mathcal{V}_0 : \mathbb{K} \times [0, \mathcal{T}_2] \rightarrow \mathbb{R}, \forall z \in \mathbb{K}$ satisfying

$$\mathcal{V}_0(z(\cdot), 0) = \mathcal{V}_0(z(\cdot), \eta_k), \quad (2.32)$$

with $0 < \eta_k \in [\mathcal{T}_1, \mathcal{T}_2]$. Define $\mathcal{W}(x(t), \tau(t)) = V(x(t)) + \mathcal{V}_0(x(t), \tau(t))$. If the inequality

$$\dot{\mathcal{W}}(x(t), \tau(t)) = \frac{d}{dt}[V(x(t)) + \mathcal{V}_0(x(t), \tau(t))] < 0, \quad (2.33)$$

holds for all $(k, \eta_k, \tau(t)) \in \mathbb{N} \times [\mathcal{T}_1, \mathcal{T}_2] \times [0, \eta_k]$, then:

- i. the increment of the Lyapunov function $V(x(t))$ between two consecutive sampling instants is strictly negative, i.e., satisfies

$$\Delta V(x(t_{k+1})) = V(x(t_{k+1})) - V(x(t_k)) < 0; \quad (2.34)$$

- ii. the sampled-data closed-loop system with sampling intervals $\eta_k \in [\mathcal{T}_1, \mathcal{T}_2]$ is asymptotically stable.

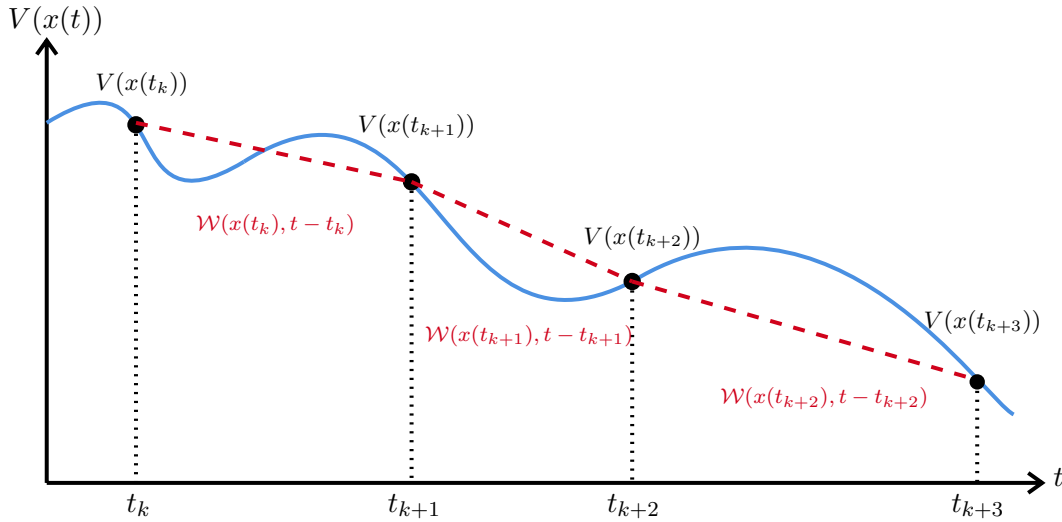


Figure 2.3: Illustration of Theorem 2.3 (Adapted from Seuret (2012)).

Figure 2.3 illustrates the behavior of Theorem 2.3. Note that, if $\mathcal{W}(x(t), \tau(t))$ is monotonically decreasing, function $V(x(t))$ can increase between the sampling instants if $\mathcal{V}_0(x(t), 0) = \mathcal{V}_0(x(t), \eta_k)$.

To illustrate a simple looped-functional, consider the following

$$\begin{aligned} V(x(t)) &= x^\top(t)Px(t), \\ \mathcal{V}_0(x(t), \tau(t)) &= (\eta_k - \tau(t)) \int_{t-\tau(t)}^t \dot{x}^\top(s)R\dot{x}(s)ds, \end{aligned} \quad (2.35)$$

Note that, considering $\mathcal{W}(x(t), \tau(t)) = V(x(t)) + \mathcal{V}_0(x(t), \tau(t))$, it follows that

$$\mathcal{W}(x(t_k), 0) = \mathcal{W}(x(t_k), \mathcal{T}_2) = \mathcal{W}(x(t_k), \eta_k) = x^\top(t_k)Px(t_k),$$

with $\eta_k \in [\mathcal{T}_1, \mathcal{T}_2]$. Therefore, if $P \in \mathbb{R}^n$ is a symmetric and positive-definite matrix, $\forall t \in [t_k, t_{k+1})$, and $\dot{\mathcal{W}}(x(t), \tau(t)) < 0$, then Theorem 2.3 is satisfied.

Several works used the looped-functional framework for the stability analysis and controller synthesis of sampled-data systems with aperiodic samplings in the context of polytopic systems

with time-varying parameters. In the LPV framework, [Gomes da Silva Jr. et al. \(2015\)](#) used the looped-LKF from [\(Seuret, 2012\)](#) to reach the stability and stabilization of sampled-data LPV systems using state feedback control, and extended the result using a parameter-dependent version considering energy-bounded disturbances ([Gomes da Silva Jr. et al., 2018](#)) and input saturation ([Palmeira et al., 2018](#)); [Hooshmandi et al. \(2018a\)](#) applied the Wirtinger's inequality in a looped-LKF to reach less conservative results; [Hooshmandi et al. \(2020\)](#) proposed the use of a piecewise smooth looped-functional with polynomial dependence on the parameters; [Han and Lee \(2021\)](#) used a time-dependent Lyapunov function approach and stability conditions in terms of sum-of-squares. Other recent results in those contexts can be found in [Jung et al. \(2021\)](#); [Wei et al. \(2022\)](#); [Palmeira et al. \(2022\)](#). Considering *quasi*-LPV systems and T-S fuzzy systems, looped-functionals had been considered in [Zhu et al. \(2012, 2013\)](#); [Wu et al. \(2014\)](#); [Cheng et al. \(2017\)](#); [Kim et al. \(2018\)](#); [Lopes et al. \(2020a\)](#); [Coutinho et al. \(2021\)](#); [Lopes et al. \(2021\)](#); [Palmeira et al. \(2021\)](#). However, except for [Lopes et al. \(2020a, 2021\)](#); [Coutinho et al. \(2021\)](#); [Palmeira et al. \(2021\)](#), the works do not consider the domain of validity of the *quasi*-LPV or T-S fuzzy models. Since the parameters depend on states of the system, it is necessary to consider the limits of validity of the model, making them valid only in a particular region of the state space, resulting in local stabilization ([Silva et al., 2020](#)). Also, in the mentioned works, except for [Lopes et al. \(2020a, 2021\)](#), the derivative of the looped-LKF results in a first-order polynomial in function of $\tau(t)$. In this way, LMI conditions can be obtained using the linearity between the minimum and maximum of the intervals. However, it is possible to derive less conservative stability criteria through quadratic functions of the time-varying delay $\tau(t)$. The following subsection provides a context for such approaches.

2.3.1 Stability Conditions as Quadratic Polynomials

Consider a looped-functional given by:

$$\begin{aligned} V(x(t)) &= x^\top(t)Px(t), \\ \mathcal{V}_0(x(t), \tau(t)) &= (\eta_k - \tau(t))\tau(t)(x(t) - x(t - \tau(t)))^\top U(x(t) - x(t - \tau(t))). \end{aligned} \quad (2.36)$$

Considering $0 < P$, conditions (2.31) and (2.32) of Theorem 2.3 are satisfied. Therefore, it remains to verify (2.33), that can be expressed as:

$$\begin{aligned} \dot{\mathcal{W}}(x(t), \tau(t)) &= \dot{x}^\top(t)Px(t) + x^\top(t)P\dot{x}(t) - \tau(t)(x(t) - x(t - \tau(t)))^\top U(x(t) - x(t - \tau(t))) \\ &\quad + (\eta_k - \tau(t))(x(t) - x(t - \tau(t)))^\top U(x(t) - x(t - \tau(t))) \\ &+ (\eta_k - \tau(t))\tau(t)\dot{x}^\top(t)^\top U(x(t) - x(t - \tau(t))) + (\eta_k - \tau(t))\tau(t)(x(t) - x(t - \tau(t)))^\top U\dot{x}(t) < 0. \end{aligned}$$

Note that $\dot{x}(t - \tau(t)) = 0$. It is important to notice that the terms

$$(\eta_k - \tau(t))\tau(t)\dot{x}^\top(t)U(x(t) - x(t - \tau(t))) \quad \text{and} \quad (\eta_k - \tau(t))\tau(t)(x(t) - x(t - \tau(t)))^\top U\dot{x}(t),$$

have quadratic parts with respect to the time-varying delay $\tau(t)$, i.e., of order $\tau^2(t)$. Therefore, the chosen looped-functional results in a second-order polynomial negative definite locally in

$\tau(t) \in [0, \eta_k]$. Then, to guarantee the stability of the sampled-data control system, it is necessary that the second order polynomial matrix in the functional satisfies:

$$\mathcal{P}(\tau(t)) = \tau^2(t)\Pi_2 + \tau(t)\Pi_1 + \Pi_0 < \mathbf{0}, \forall \tau(t) \in [\underline{\tau}, \bar{\tau}], \quad (2.37)$$

where $\Pi_i \in \mathbb{S}^n$ ($i = 0, 1, 2$) and $\tau(t)$ is a real bounded scalar quantity. Note that Π_i ($i = 0, 1, 2$) contain the decision variables that need to be obtained to guarantee the negative definiteness of the polynomial. Therefore, it is not trivial to give sharp LMI conditions (i.e., the less conservative possible) to ensure the stability of the closed-loop system.

Such constraints play an important role in control theory, especially for stability analysis of dynamic systems with time-varying delays (see, e.g., Zhang et al. (2022b); Chen et al. (2022b)). However, since these matrix inequalities are not convex in $\tau(t)$, ongoing research focuses on providing conditions based on linear matrix inequality (LMI) to satisfy (2.37) with minimal conservatism. Widely used approaches involve geometrical considerations to establish sufficient conditions, summarized in the following lemmas, which include the pioneer and most recent related results from the literature; see, e.g., Kim (2011); Yang and Zhang (2014); Kim (2016); Yang et al. (2019); Chen et al. (2019); Zhang et al. (2020a); Liu et al. (2023) and the surveys in Zhang et al. (2022b); Chen et al. (2022b); He et al. (2023) for a complete overview. It follows from (2.37) that:

$$\mathcal{P}_\xi(\tau(t)) = \xi^\top \mathcal{P}(\tau(t)) \xi = \tau^2(t)a_2 + \tau(t)a_1 + a_0 < 0, \quad (2.38)$$

where $a_i = \xi^\top \Pi_i \xi \in \mathbb{R}$, ($i = 0, 1, 2$), $\forall \xi \in \mathbb{R}^n / \{0\}$. Hence, sufficient conditions for (2.37) arise from considering the convexity or concavity of the second-order polynomial (2.38). From this point of view, sufficient conditions were proposed in Kim (2011) (or equivalently in Yang and Zhang (2014)) by assuming that $\mathcal{P}_\xi(\tau(t))$ is convex, i.e., $a_2 \geq 0$ (or equivalently $\Pi_2 \geq \mathbf{0}$ in (2.37)). However, this requirement can be very restrictive and conservative. As shown in

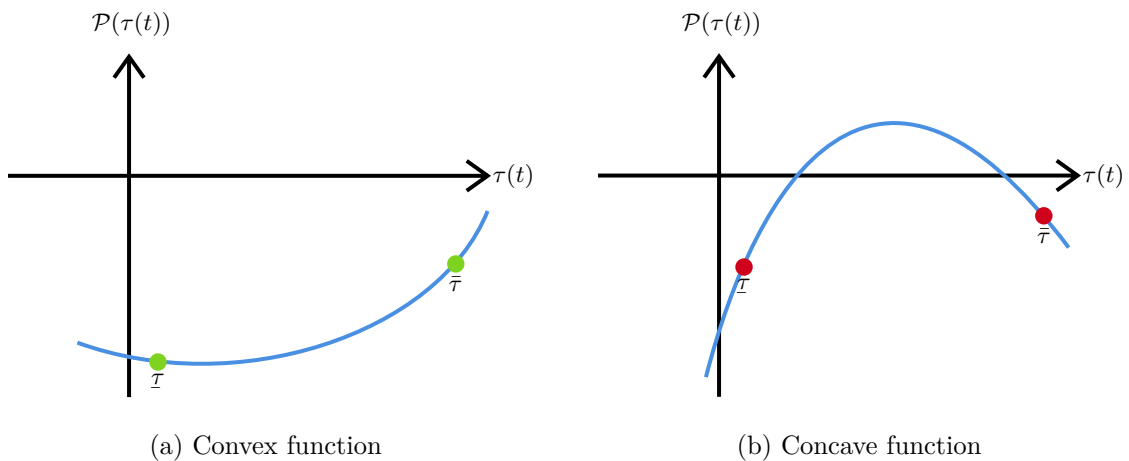


Figure 2.4: Geometrical interpretation of the approach proposed in Kim (2011). In (a), a convex function is tested in the green dots, resulting in a necessary and sufficient condition; In (b), it can be noted that the method is not sufficient for cases of concave functions since the function can be positive in the interval even if the values tested (red dots) are not.

Figure 2.4, checking the negativity of the function at the minimum and maximum points of

$\tau(t)$ is a necessary and sufficient condition to guarantee the negativity of function (2.38) if it is convex. However, the test is insufficient for cases where the function is concave, resulting in a very conservative condition.

Hence, relaxed conditions were proposed to cope with either convex or concave second order polynomials. Less conservative conditions were proposed, among others, in Kim (2016); Yang et al. (2019). These have been further improved in Zhang et al. (2020a) and more recently in Liu et al. (2023), where an extra parameter, usually tuned by linear search (otherwise, the conditions could not be convex), is introduced. It should be emphasized that these primary geometrical conditions are only sufficient. Hence, partitioning approaches can be considered to further relax them by splitting the interval of the definition of $\tau(t)$ into $N_p \in \mathbb{N}^*$ partitions i.e., $[\underline{\tau}, \bar{\tau}] = \cup_{i=1}^{N_p} [\underline{\tau}_i, \bar{\tau}_i]$ with $\bar{\tau}_i = \underline{\tau}_{i+1}$, $\underline{\tau}_1 = \underline{\tau}$, and $\bar{\tau}_{N_p} = \bar{\tau}$, as recently proposed in Chen et al. (2019); He et al. (2023); Liu et al. (2023). Note that the variable N_p refers to the number of partitions of the $\tau(t)$ interval. Figure 2.5 illustrates a geometrical interpretation of two approaches that

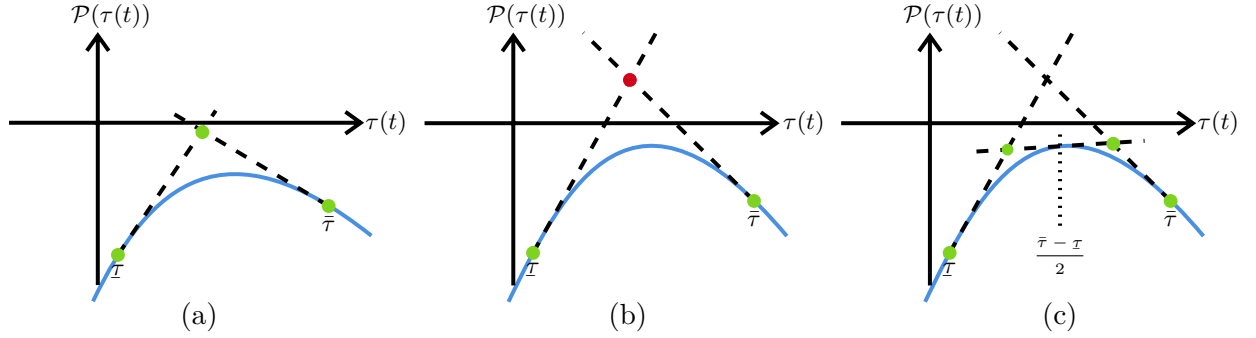


Figure 2.5: (a) Geometrical interpretation of the lemma presented in Park and Park (2020): tangent lines at the maximum and minimum points of the function in the interval $[\underline{\tau}, \bar{\tau}]$ are drawn, so if the meeting point between them is negative, it is sufficient to guarantee that the function is negative. Thus, concave functions can be tested by the green dots; (b) A different function is tested with the previous lemma. However, the red dot is not negative, demonstrating the conservatism in defining the polynomial negativity; (c) The proposal of Liu et al. (2023) is illustrated. The interval of $\tau(t)$ is divided, and a new tangent is traced over this point, then the meeting points are tested in the green dots, reducing the conservatism.

evaluate the negativity of a polynomial presented in the literature.

The partitioning approaches provide a recursive relaxation to guarantee the negative definiteness of quadratic polynomials. However, when a solution holds for a given N_p , there is no guarantee that a solution holds for $N_p + 1$ partitions (more details about this are given in Chapter 4). Therefore, these geometrical conditions with a partitioning approach are non-monotonic convergent (non-MC) as far as N_p can be enlarged. This fact can be understood as a drawback of such partitioning approaches since it can be difficult to search for a large enough N_p to approximate the optimal solution.

Besides the extensive research efforts to obtain less conservative conditions based on geometrical methods, recent studies were also conducted to provide necessary and sufficient conditions applying the generalized S-procedure (de Oliveira and Souza, 2020; Chen et al., 2022a; Park and

Park, 2020; Kim, 2021). However, these works require the introduction of additional decision variables. As claimed by many authors of recent studies (see, e.g., Zeng et al. (2020); Chen and Li (2021); Zhang et al. (2022b); Chen et al. (2022b); He et al. (2023); Xie et al. (2023)), such requirements can be extensively demanding and may lead to a computational burden, especially for large-sized matrix inequalities which already involves a massive number of decision variables, as often encountered in the time-delay framework (for instance when they are derived from Jensen, Wirtinger or Bessel-Legendre inequalities (Seuret and Gouaisbaut, 2015; Liu et al., 2017; Kim, 2021; Lopes et al., 2021; Rouamel et al., 2023)).

Due to these drawbacks of the necessary and sufficient approaches, it remains challenging to establish further monotonic convergent conditions that both avoid introducing extra slack decision variables and improve upon previous geometrical methods, while still reaching the optimal solution. Moreover, all the aforementioned works focus exclusively on precisely known models, i.e., when Π_i ($i = 0, 1, 2$) in (2.37) are independent of any time-varying or uncertain parameter. Consequently, even the necessary and sufficient conditions have not been investigated for the stability analysis and controller design of LPV systems. It is worth noting that, in such cases, S-procedure-based methods are no longer necessary and remain only sufficient. Chapter 4 provides an overview of the main approaches and introduces three new methods for addressing the negative definiteness of quadratic polynomials, particularly for time-varying parameter cases, thereby yielding monotonic convergence conditions.

2.4 Input Saturation

Actuator saturation is widely present in control systems. Several strategies have been adopted over the years to overcome the undesirable effects of this type of nonlinearity in closed-loop systems. In particular, in the case of sampled-data systems, the stability and stabilization of precisely known systems with rate and magnitude saturation were investigated in Palmeira et al. (2016).

On the other hand, contributions to sampled-data systems that present saturation only in magnitude have grown in recent years. In the framework of precisely known systems, actuators saturating in magnitude were investigated in sampled-data control with the looped-LKF approach (Seuret and Gomes Da Silva Jr., 2012), in the representation of sampled-data systems as impulsive systems (Fiacchini and Gomes Da Silva Jr., 2018; Huff et al., 2022b,a), and in the form of hybrid systems (Fagundes and Gomes Da Silva Jr., 2022; Fagundes et al., 2024a). In the polytopic time-varying systems framework, the looped-LKF approach is used in the stability and stabilization of LPV systems through a model predictive control (MPC) in Shi and Su (2014). Likewise, Palmeira et al. (2018) proposes state feedback control in the context of LPV systems and Lopes et al. (2021) for T-S Fuzzy systems, dealing with the saturation of actuators under the generalized sector condition.

To date, no prior studies in the literature have addressed actuator saturation in both rate and magnitude for LPV systems using sampled-data control, which is one of the objectives of this

work. In the following subsections, the theory of the generalized sector condition is presented in the context of magnitude saturation, which is also used to model rate saturation. This framework is applied in Chapter 3 to design parameter-dependent controllers for LPV discrete-time systems, and in Chapter 6 to design parameter-dependent controllers for the sampled-data control of LPV systems.

2.4.1 Magnitude Saturation

Classically, there are three main approaches to model the presence of magnitude saturation in control systems in the literature. The first is based on modeling saturation as a dead zone nonlinearity and is called the generalized sector condition, proposed by [Gomes da Silva Jr. and Tarbouriech \(2005\)](#). The second approach describes the saturated system in a polytopic model and presents three types. The so-called polytopic model I was introduced by [Molchanov and Pyatnitskii \(1989\)](#), the polytopic model II was suggested by [Hu and Lin \(2001\)](#), and the model III was initially proposed in ([Alamo et al., 2006](#)). These denominations are found in ([Tarbouriech et al., 2011](#), pp. 33), where the reader can look for more details. The main disadvantage of the polytopic approach is the increasing numerical complexity in optimization procedures, which becomes even more critical in polytopic systems. The third approach to dealing with saturating inputs is derived from the MPC control strategy, in which the outputs at future instants of time and the control law are calculated through an optimization procedure at each instant ([Wang, 2009](#)). Recently, [Groff et al. \(2019\)](#) proposed a model with implicit piecewise affine functions to treat saturating actuators in discrete-time systems. One of the advantages of this method is the possibility of considering asymmetric saturation limits. These conditions were developed in the context of local stability in [Cabral et al. \(2025\)](#).

To demonstrate the approach using the generalized sector condition applied in Chapter 3 and Chapter 6, recall the continuous-time parameter-dependent system in (2.1) with saturation inputs:

$$\begin{aligned} \dot{x}(t) &= A(\alpha(t))x(t) + B(\alpha(t))\text{sat}(u(t)), \\ y(t) &= C(\alpha(t))x(t) \end{aligned} \tag{2.39}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is the control signal vector, $A(\alpha(t)) \in \mathbb{R}^{n \times n}$, $B(\alpha(t)) \in \mathbb{R}^{n_u \times n}$, and $C(\alpha(t)) \in \mathbb{R}^{n_y \times n}$ depend linearly on the time-varying parameter vector $\alpha(t)$ belonging to polytopic domain given in (2.2) with $\alpha(t) \in \Gamma_N$.

The system has input magnitude bounds, given by $\mu_{(\ell)} \in \mathbb{R}^{n_u}$, where the saturation function is described as

$$\text{sat}(u_{(\ell)}(t)) = \begin{cases} \mu_{(\ell)}, & \text{if } u_{(\ell)}(t) > \mu_{(\ell)} \\ u_{(\ell)}(t), & \text{if } -\mu_{(\ell)} \leq u_{(\ell)}(t) \leq \mu_{(\ell)} \\ -\mu_{(\ell)}, & \text{if } u_{(\ell)}(t) < -\mu_{(\ell)} \end{cases} \tag{2.40}$$

and $\mu_{(\ell)}$, $\ell = 1, \dots, n_u$, is the symmetrical magnitude bound for each saturating actuator.

The control law applied to the system, considering a time-varying parameter-dependent controller, is given by

$$u(t) = K(\alpha(t))x(t), \tag{2.41}$$

where $K(\alpha(t)) \in \mathbb{R}^{n_u \times n}$ belong to a polytopic domain given by (2.2) with $\alpha(t) \in \Gamma_N$. Note that the control signal is limited in magnitude, and the closed-loop system can be written by substituting (2.41) in (2.39), resulting in

$$\begin{aligned}\dot{x}(t) &= A(\alpha(t))x(t) + B(\alpha(t))\text{sat}(K(\alpha(t))x(t)), \\ y(t) &= C(\alpha(t))x(t)\end{aligned}\tag{2.42}$$

Hence, the control signal available in system (2.42) for an input ℓ is shown in Figure 2.6, where the nonlinear behavior of the saturation function can be observed.

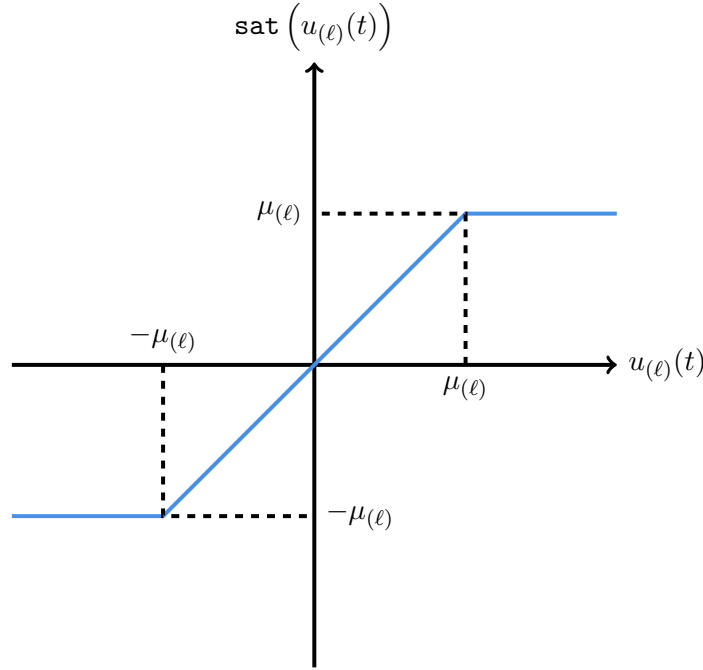


Figure 2.6: Saturation function.

Due to the nonlinear constraints in the actuator, global closed-loop stability generally cannot be ensured. Thus, it is necessary to determine the region of attraction $\mathcal{R}_a \subseteq \mathbb{R}^n$ of all initial conditions such that the corresponding trajectories of (2.42) converge to the origin. Because $\mathcal{R}_a$ can be non-convex, open, and even unbounded in some directions (Tarbouriech et al., 2006), an estimate $\mathcal{R}_{\mathcal{E}_a} \subseteq \mathcal{R}_a$ is calculated as large as possible. In the literature, this estimate is made through several approaches (Blanchini, 1999). Among them, the most common are through ellipsoidal (Jungers and Castelan, 2011; Seuret and Gomes Da Silva Jr., 2012; Huff et al., 2022b) and polyhedral (Olaru and Niculescu, 2008; Hempel et al., 2011; Fiacchini and Morarescu, 2016; Huff et al., 2022a) sets. Thus, the controller designed considering the effect of saturation guarantees the local asymptotic stability of system (2.42). The saturation nonlinearity is written as a dead-zone nonlinearity, defined by

$$\Psi(u(t)) = \text{sat}(u(t)) - u(t),\tag{2.43}$$

that is, $\Psi(u(t)) = [\Psi_1(u(t)), \dots, \Psi_{n_u}(u(t))]^\top$.

Replacing the dead-zone function (2.43) in (2.42), the closed-loop system can be written as

$$\begin{aligned} \dot{x}(t) &= \left(A(\alpha(t)) + B(\alpha(t))K(\alpha(t)) \right) x(t) + B(\alpha(t))\Psi\left(K(\alpha(t))x(t)\right), \\ y(t) &= C(\alpha(t))x(t). \end{aligned} \quad (2.44)$$

Therefore, the system can be seen as a Lur'e type system, characterized by a linear part and a memoryless nonlinearity satisfying a sector condition in its dynamics (Tarbouriech et al., 2011, pp. 40). Therefore, the generalized sector condition can be employed to establish stability conditions for systems subject to actuator saturation.

Consider the signal $u(t) - v(t)$ where the auxiliary signal $v(t) = Gx(t)$ is used as a degree of freedom in the controller design. Therefore, the set $\mathbb{S}$ is defined such that $u(t) - v(t)$ belong to:

$$\mathbb{S} = \left\{ u(t) \in \mathbb{R}^{n_u}, v(t) \in \mathbb{R}^{n_u} : \left| u_{(\ell)}(t) - v_{(\ell)}(t) \right| \leq \mu_{(\ell)} \right\}, \quad (2.45)$$

for all $\ell = 1 \dots, n_u$. Then, the following lemma presented by Gomes da Silva Jr. and Tarbouriech (2005) can be stated:

Lemma 2.3 (Generalized sector condition). *Consider the function $\Psi(u(t))$ defined in (2.43). If $x(t) \in \mathbb{S}$, then the relation*

$$\Psi^\top(u(t))T(\Psi(u(t)) + v(t)) \leq 0 \quad (2.46)$$

is verified for any positive definite diagonal matrix $T \in \mathbb{R}^{n_u \times n_u}$.

Note that, for the stabilization of system (2.44), the gain matrix $K(\alpha(t))$ is parameter-dependent. Then, the extra degree of freedom signal $v(t) = G(\alpha(t))x(t)$ can be considered with a similar structure to the control signal $u(t)$, depending linearly on $\alpha(t)$. An extended version of this lemma, considering magnitude and rate saturation, is presented in Chapter 3 for discrete-time systems and in Chapter 6 for sampled-data control systems.

2.4.2 Magnitude and Rate Saturation

In handling saturating actuators in (slew-)rate and magnitude, two main methods have been proposed (Baiomy and Kikuuwe, 2020). The first one involves designing a controller that incorporates the nonlinear actuator model, ensuring that the control signal does not exceed the rate and magnitude limits of the actuator. For continuous-time systems, conditions based on such a method are presented in Kapila and Haddad (1998) and Kapila et al. (1999). Similarly, in Pan and Kapila (2001) for discrete-time systems, the basic sector condition (Tarbouriech et al., 2011) helps to represent the saturation nonlinearity phenomenon, allowing a convex formulation for stability analysis. A method to design a dynamic controller with anti-windup action and integrators has been proposed in Gomes da Silva Jr. et al. (2008) for discrete-time systems using the generalized sector condition (Gomes da Silva Jr. and Tarbouriech, 2005). In such a work, the authors also seek to maximize the size of the region of attraction to ensure the local asymptotic stability of the closed-loop system. A similar approach to ensure the internal and external stability of continuous-time systems subject to ℓ_2 disturbances is addressed in Bender

and Gomes da Silva Jr. (2012). Sliding mode-based controllers for continuous and discrete-time systems, while considering rate-saturating actuators, are investigated in Baiomy and Kikuuwe (2020). Additional works in continuous-time systems include Galeani et al. (2008) and Forni et al. (2012). In these works, the system control affected by rate and magnitude saturating actuators is improved with an anti-windup action.

The second method, known as the position-type feedback model with speed limitation (PMSL), employs a first-order system with a magnitude-saturating input signal and the rate saturation modeled as a magnitude constraint on the actuator state. In such a case, nested saturations are obtained, which requires a more involved mathematical approach (Tarbouriech et al., 2006).

Tyan and Bernstein (1997) design a dynamic compensator to address independent rate and magnitude saturation. In contrast, semi-global and local stabilization methods are proposed in Lin (1997) and Gomes da Silva Jr. et al. (2003), respectively. General conditions for systems with nested saturations are given in Bateman and Lin (2003) for continuous-time systems and Bateman and Lin (2002); Zhou (2013) for discrete-time systems. These methods involve the utilization of the polytopic model for saturating signals. It is worth noting that the generalized sector condition is used in Tarbouriech et al. (2006) to handle nested saturation in continuous-time systems. Consequently, the results are usually less conservative than those in Kapila and Haddad (1998) and Kapila et al. (1999). Also, considering nested saturation, the authors in Palmeira et al. (2016) provide a design method for stabilizing sampled-data control systems under magnitude and rate saturation in the actuator. Thus, the case of LPV systems with saturating actuators in both magnitude and rate has not yet been investigated in sampled-data control systems. In Oliveira et al. (2022), the class of discrete-time LPV systems under rate and magnitude saturation is investigated, and a convex design method is proposed. However, the effects of disturbances on closed-loop trajectories are not considered.

Recent works in such a context include Ernesto et al. (2022), where authors introduce an output feedback controller design method based on positively polyhedra obtained by bilinear optimization for discrete-time systems. In Kucukdemiral (2020) and Kucukdemiral and Yazici (2021), researchers design robust gain-scheduled controllers using a modified full-block S-procedure method for uncertain discrete-time and continuous-time systems with bounded disturbances. Using the same approach, the authors propose an extension for robust output and state feedback control of uncertain discrete-time systems in Kucukdemiral et al. (2022).

In the works of this thesis, the PMSL is considered to model the magnitude and rate saturation in the actuators. Therefore, it is possible to address both nonlinearities using the generalized-sector condition. To demonstrate the effects of using such a model, consider a continuous-time system modeled as an LPV system defined as:

$$\dot{x}_s(t) = A(\alpha(t))x_s(t) + B(\alpha(t))\Phi(u(t)), \quad (2.47)$$

where $x_s \in \mathbb{R}^n$ is the state of the system and $\Phi(u(t)) \in \mathbb{R}^{n_u}$ is the input delivered by the actuator. $A(\alpha(t)) \in \mathbb{R}^{n \times n}$ and $B(\alpha(t)) \in \mathbb{R}^{n_u \times n}$ depend linearly on the time-varying parameter

vector $\alpha(t)$ belonging to the polytopic domain given in (2.2) with $\alpha(t) \in \Gamma_N$. Hence, $\Phi(u(t))$ must contain both magnitude and rate saturating aspects of the actuator. The PMSL, presented in Figure 2.7, can model both saturations in each actuator and has the dynamics given by:

$$\dot{x}_{a(\ell)}(t) = \text{sat}_{\mathbf{R}(\ell)} \left(\lambda_{(\ell)} \text{sat}_{\mathbf{M}(\ell)} \left(u_{(\ell)}(t) \right) - \lambda_{(i)} x_{a(\ell)}(t) \right), \quad (2.48)$$

for $\ell = 1, \dots, n_u$, with $x_a \equiv \Phi(u(t)) \in \mathbb{R}^{n_u}$ and $u \in \mathbb{R}^{n_u}$ stand for the state of the actuator and the control input, respectively. The saturation functions of magnitude and rate are given respectively by $\text{sat}_{\mathbf{M}}(u_{(\ell)}(t)) = \text{sign}(u_{(\ell)}(t)) \min\{|u_{(\ell)}(t)|, \mu_{\mathbf{M}(\ell)}\}$ and $\text{sat}_{\mathbf{R}}(u_{(\ell)}(t)) = \text{sign}(u_{(\ell)}(t)) \min\{|u_{(\ell)}(t)|, \mu_{\mathbf{R}(\ell)}\}$, where the positive elements $\mu_{\mathbf{M}(\ell)}$ and $\mu_{\mathbf{R}(\ell)}$ define the saturation limits of each actuator. The parameter $\lambda_{(\ell)} > 0$ regards the actuator dynamics, and when chosen as $\lambda \rightarrow \infty$, it approximates the ideal rate limiter. Assuming the saturation limits $\mu_{\mathbf{M}} = 10$

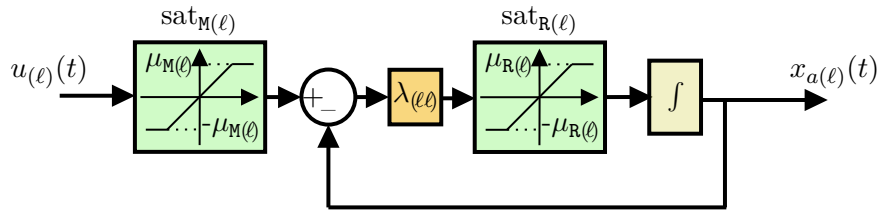


Figure 2.7: Diagram of the continuous-time PMSL used to model magnitude and rate saturation for an input ℓ .

and $\mu_{\mathbf{R}} = 0.2$, Figure 2.8 illustrates the output of the PMSL in comparison with an ideal rate limiter for different values of λ for an input ℓ .

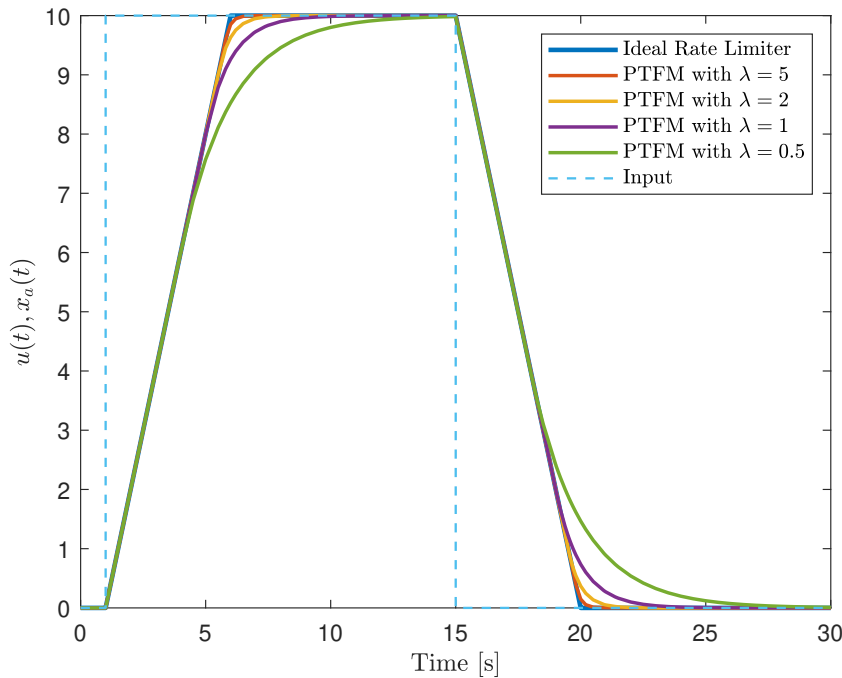


Figure 2.8: Step response of the continuous-time PMSL for an input ℓ and different values of λ .

Now, taking a discrete-time LPV model as

$$x_s(k+1) = A(\alpha(k))x_s(k) + B(\alpha(k))\Phi(u(k)), \quad (2.49)$$

where $x_s \in \mathbb{R}^n$ is the state of the system and $\Phi(u(k)) \in \mathbb{R}^{n_u}$ is the output of the actuator. $A(\alpha(k)) \in \mathbb{R}^{n \times n}$ and $B(\alpha(k)) \in \mathbb{R}^{n_u \times n}$ depend linearly on the time-varying parameter vector $\alpha(k)$ belonging to the polytopic domain given in (2.2) with $\alpha(k) \in \Gamma_N$. Then, the discrete-time version of the PMSL, illustrated in Figure 2.9, is given by:

$$x_{a(\ell)}(k+1) = x_{a(\ell)}(k) + \text{sat}_{R(\ell)} \left(\lambda_{(\ell)} \text{sat}_{M(\ell)} \left(u_{(\ell)}(k) \right) - \lambda_{(i)} x_{a(\ell)}(k) \right), \quad (2.50)$$

for $\ell = 1, \dots, n_u$. In the discrete-time framework, the parameter $\lambda_{(\ell)} > 0$ approximates the ideal rate limiter when chosen as $\lambda \rightarrow 1$. For a sampling time $T_s = 0.5$ seconds, and assuming the saturation limits $\mu_M = 10$ and $\mu_R = 0.2$, Figure 2.10 illustrates the output of the discrete-time PMSL in comparison with an ideal rate limiter for different values of λ considering a single input ℓ .

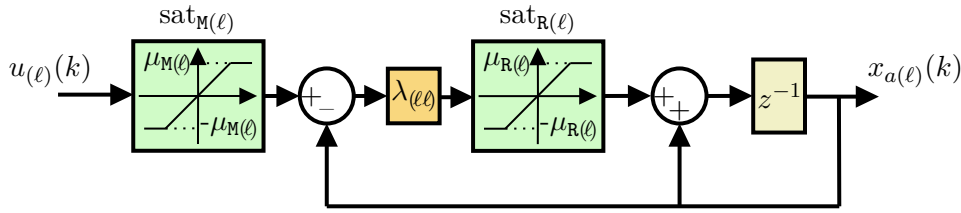


Figure 2.9: Diagram of the discrete-time PMSL used to model magnitude and rate saturation for an input ℓ .

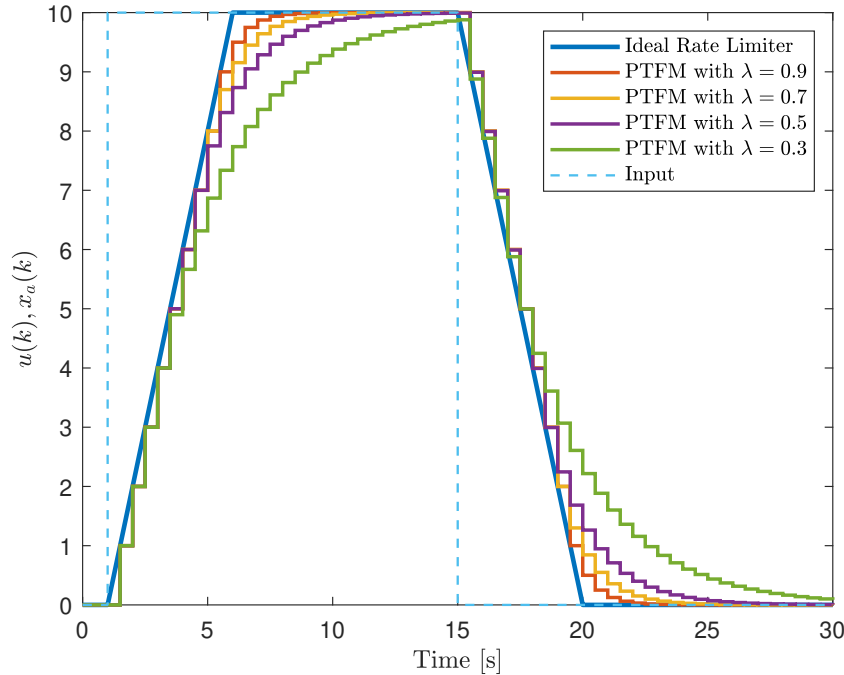


Figure 2.10: Step response of the discrete-time PMSL for an input ℓ and different values of λ .

Both models, the continuous-time PMSL and the discrete-time PMSL, are used in Chapter 3 and Chapter 6 to address, respectively, the stabilization of LPV discrete-time systems subject to amplitude disturbances and the stabilization of LPV sampled-data control systems. The use of sampled-data controllers makes it possible to design a discrete-time controller without first deriving a discrete-time model of the system, enabling flexible control-update intervals and offering more favorable stability conditions in scenarios involving aperiodic sampling instants and network-induced adversities in the control loop.

2.5 Final Considerations

This chapter established the essential theoretical foundations that support the contributions of this thesis. The concepts of parameter-dependent systems, such as LPV and T-S Fuzzy, were reviewed, highlighting their capability to model a wide range of processes with time-varying parameters. The sampled-data control approach was introduced as a robust solution, modeled through the perspective of input-delay systems to handle aperiodic sampling intervals.

Lyapunov stability theory was presented as the main stability analysis tool, with an emphasis on the use of Lyapunov-Krasovskii functionals and the looped-functionals technique to reduce conservatism. Furthermore, the practical challenges imposed by actuator saturation, in both magnitude and rate, were discussed, presenting the generalized sector condition and the PMSL model as the adopted strategies to handle these nonlinearities.

Finally, the chapter outlined the main challenges to be addressed, such as obtaining less conservative stabilization conditions in terms of LMIs. With this theoretical basis consolidated, the following chapters are dedicated to developing and presenting novel solutions to these problems.

Stabilization of Discrete-Time LPV Systems with Rate and Magnitude Saturating Actuators

Tom said to himself that it was not such a hollow world, after all. He had discovered a great law of human action, without knowing it — namely, that in order to make a man or a boy covet a thing, it is only necessary to make the thing difficult to attain. If he had been a great and wise philosopher, like the writer of this book, he would now have comprehended that Work consists of whatever a body is obliged to do, and that Play consists of whatever a body is not obliged to do. And this would help him to understand why constructing artificial flowers or performing on a tread-mill is work, while rolling ten-pins or climbing Mont Blanc is only amusement.

The Adventures of Tom Sawyer (1876)
MARK TWAIN

This chapter tackles the critical challenge of designing parameter-dependent controllers for the wide class of discrete-time time-varying systems under rate and magnitude saturating actuators. Such a class finds widespread use in diverse applications, from medicine to industrial engineering. The main contribution lies in developing novel convex parameter-dependent state feedback controller synthesis conditions that ensure regional stability, effectively mitigating the adverse effects of rate and magnitude-saturating actuators. The proposed approach considers the ℓ_2/ℓ_∞ gain between the disturbance input and the controlled output, providing performance specifications and remaining valid for specific initial conditions and amplitude-limited input disturbances. Moreover, the methodology is highly adaptable and suitable for a broad spectrum of systems, including LPV, *quasi*-LPV, and T-S fuzzy systems. The position-type feedback model with speed limitation and the generalized sector condition is leveraged to derive the controller synthesis method. This results in a parameter-dependent compensation strategy under standard assumptions, ensuring practicality and applicability to diverse system requirements. The effectiveness of the proposed approach is demonstrated through numerical examples compared with existing literature and validated by real-time experiments on a nonlinear coupled tank system, highlighting its efficacy and feasibility for real-world implementation.

3.1 Problem Formulation

The class of systems treated in this chapter concerns nonlinear discrete-time systems subject to rate and magnitude saturating actuators and amplitude-bounded disturbances described by:

$$\begin{aligned} x(k+1) &= f(x(k)) + B(x(k))\Phi(u(k)) + B_\omega(x(k))\omega(k), \\ z(k) &= g(x(k)) + D(x(k))\omega(k). \end{aligned} \quad (3.1)$$

where $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$, $g : \mathbb{R}^n \rightarrow \mathbb{R}^{n_z}$, $B : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n_u}$, $B_\omega : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n_\omega}$, and $D : \mathbb{R}^n \rightarrow \mathbb{R}^{n_z \times n_\omega}$ are nonlinear functions depending on the states, $x(k) \in \mathbb{R}^n$ is the state vector, $\Phi(u(k)) \in \mathbb{R}^{n_u}$ is the input delivered by the actuator, $z(k) \in \mathbb{R}^{n_z}$ is the performance output, and $\omega(k) \in \mathbb{R}^{n_\omega}$ is an ℓ_∞ exogenous signal at instant k belonging to the following set:

$$\mathcal{W} = \{\omega \in \mathbb{R}^{n_\omega} : \omega(k)^\top \omega(k) \leq \delta^{-1}, \forall k \in \mathbb{N}\}, \quad (3.2)$$

$\delta > 0$, with $\sqrt{\delta^{-1}} \in \mathbb{R}_+$ representing the maximum amplitude of the disturbance signals. Observe that $\omega(k)$ captures a wide class of amplitude-bounded signals that may be stochastic, deterministic, persistent, or limited over time.

A state feedback control law is considered, in which the computed control signal $u(k)$ acts on a saturating actuator, which delivers a signal $\Phi(u(k))$ to the controlled system. Figure 3.1 presents a diagram that illustrates such an interconnection. Moreover, Figure 3.1 highlights the actuator modeling and the feedback control structure. The following sections discuss these structures and the closed-loop interconnection.

The vector-valued function $\Phi(u(k)) : \mathbb{R}^{n_u} \rightarrow \mathbb{R}^{n_u}$ represents the rate and magnitude saturation imposed by the actuators and can be interpreted as a discrete-time version of the PMSL used in [Tyan and Bernstein \(1997\)](#). From the actuator block in Figure 3.1 one gets:

$$\hat{x}(k+1) = \hat{x}(k) + \text{sat}_R(\Lambda \text{sat}_M(u(k)) - \Lambda \hat{x}(k)), \quad (3.3)$$

$$\Phi(u(k)) = \hat{x}(k), \quad (3.4)$$

where $\hat{x}(k) \in \mathbb{R}^{n_u}$ is the state of the actuator, the elements of the diagonal matrix $\Lambda \in \mathbb{R}^{n_u \times n_u}$ are related to the dynamics of the actuators, and for a control signal $u(k) \in \mathbb{R}^{n_u}$ the symmetric saturation functions of rate and magnitude are written as:

$$\text{sat}_R(u_{(\ell)}(k)) = \text{sign}(u_{(\ell)}(k)) \min(|u_{(\ell)}(k)|, \mu_{R(\ell)})$$

and

$$\text{sat}_M(u_{(\ell)}(k)) = \text{sign}(u_{(\ell)}(k)) \min(|u_{(\ell)}(k)|, \mu_{M(\ell)}),$$

for the symmetrical limits $\mu_R \in \mathbb{R}^{n_u}$ and $\mu_M \in \mathbb{R}^{n_u}$, respectively. Thus, the saturated control signal belongs to the interval $[-\mu_M, \mu_M]$, i.e., $-\mu_M \leq \Phi(u(k)) \leq \mu_M$, while the signal change between two consecutive samplings is constrained to $[-\mu_R, \mu_R]$, meaning that $-\mu_R \leq \Phi(u(k+1)) - \Phi(u(k)) \leq \mu_R$.

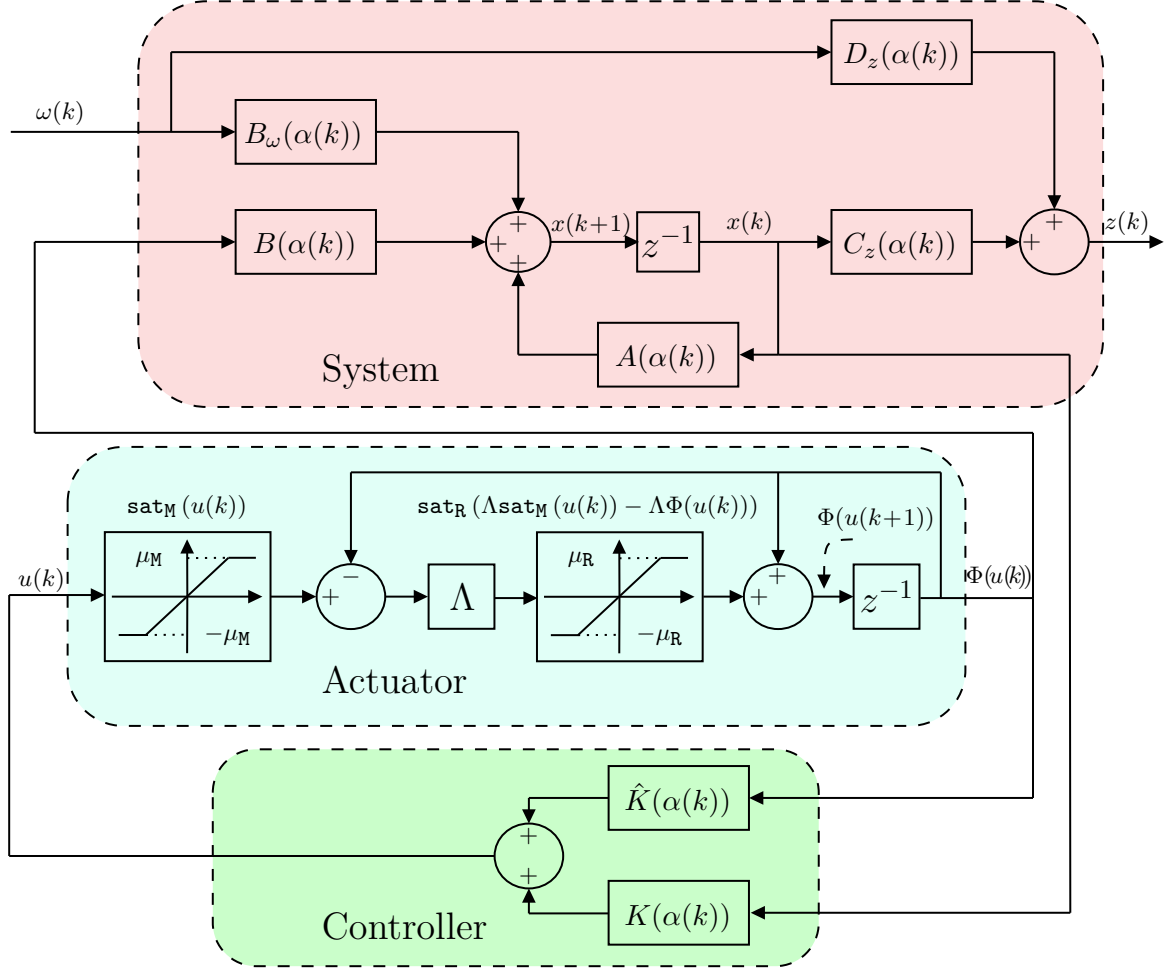


Figure 3.1: Schematic diagram of the closed-loop system where the main blocks are highlighted: the controlled system (upper), actuator (middle), and the controller (bottom).

Therefore, an ℓ_2/ℓ_∞ gain can be associated between the behavior (controlled) output and the disturbance input to address the closed-loop performance in terms of disturbance attenuation, given by:

$$\gamma = \sup_{w(k) \neq 0} \frac{\|z(k)\|_2}{\|w(k)\|_2}. \quad (3.5)$$

Thus, a $\gamma > 0$ verifying (3.5) is an upper bound ensuring a minimal mitigation degree of the disturbance signal on the performance output signal. In this sense, designing a control signal minimizing the γ value ensures a more efficient closed-loop system in terms of the ℓ_2/ℓ_∞ gain.

Remark 3.1. Note that, due to the saturating actuators, the controller design approach generally provides local closed-loop stabilization properties. However, if the open-loop system is asymptotically stable, global stabilization may be achieved, but with potential performance degradation. Consequently, it is reasonable to assume that the state of the nonlinear system (3.1)-(3.4) belongs to a valid subset of the state space given by

$$\mathbb{S}_x = \{x(k) \in \mathbb{R}^n; |L_{(r)}x(k)| \leq \rho_{(r)}, r = 1, \dots, r_c\} \quad (3.6)$$

with $\rho_{(r)} > 0$, $L_{(r)} \in \mathbb{R}^{1 \times n}$, where r_c is the number of constraints.

Hence, assuming $x(k)$ belongs to a valid subset of $\mathbb{R}^n$ where the modeling is verified, i.e., for all $x(k) \in \mathbb{S}_x$, and applying the sector nonlinearity approach [Tanaka and Wang \(2001\)](#), the nonlinear system (3.1)-(3.4) can be exactly represented, inside the valid domain of the model, by the following compact polytopic model:

$$\begin{aligned} x(k+1) &= A(\alpha(k))x(k) + B(\alpha(k))\Phi(u(k)) + B_w(\alpha(k))\omega(k), \\ z(k) &= C_z(\alpha(k))x(k) + D_z(\alpha(k))\omega(k), \end{aligned} \quad (3.7)$$

where the weighting function $\alpha(k)$ may depend on the state vector, which is supposed to be available or measured online. Moreover, the time-varying parameters belong to the unit simplex

$$\Gamma_N = \left\{ \delta \in \mathbb{R}^N : \sum_{i=1}^N \delta_i = 1, \delta_i \geq 0, i = 1, \dots, N \right\} \quad (3.8)$$

where N denotes the number of vertices of the polytopic representation. Furthermore, let Ω denote the sequence of values assumed by the vector-valued function $\alpha(k) \in \mathcal{P}$ from an initial instant k_0 :

$$\Omega = \{\alpha(k_0), \alpha(k_0 + 1), \dots\}. \quad (3.9)$$

Note that the sequence Ω may depend on the state of the system, which is the case for *quasi*-LPV ([Shamma and Cloutier, 1993](#)) or T-S fuzzy models ([Takagi and Sugeno, 1985](#)), casting the system as a set of N local (non)linear models weighted by time-varying functions, or it can assume any sequence as it occurs in the LPV modeling ([Shamma, 2012](#)). Moreover, without loss of generality, it can be assumed that $k_0 = 0$ in (3.9). Therefore, the matrices $A(\alpha(k)) \in \mathbb{R}^{n \times n}$, $B(\alpha(k)) \in \mathbb{R}^{n \times n_u}$, $B_w(\alpha(k)) \in \mathbb{R}^{n \times n_w}$, $C_z(\alpha(k)) \in \mathbb{R}^{n_z \times n}$, and $D_z(\alpha(k)) \in \mathbb{R}^{n_z \times n_w}$ exist within a polytopic domain defined as the convex combination of N known vertices:

$$\begin{bmatrix} A(\alpha(k)) & B(\alpha(k)) & B_w(\alpha(k)) & C_z(\alpha(k)) & D_z(\alpha(k)) \end{bmatrix} = \sum_{i=1}^N \alpha_i(k) \begin{bmatrix} A_i & B_i & B_{wi} & C_{zi} & D_{zi} \end{bmatrix}. \quad (3.10)$$

Considering the controller block in Figure 3.1, the following state feedback control law is proposed

$$u(k) = K(\alpha(k))x(k) + \hat{K}(\alpha(k))\hat{x}(k) = \mathbb{K}(\alpha(k))\xi(k), \quad (3.11)$$

where the controller gain matrices $K(\alpha(k)) \in \mathbb{R}^{n_u \times n}$ and $\hat{K}(\alpha(k)) \in \mathbb{R}^{n_u \times n_u}$ depend on the weighing function $\alpha(k)$, and also belong to a polytopic domain, such that

$$\mathbb{K}(\alpha(k)) = \begin{bmatrix} K(\alpha(k)) & \hat{K}(\alpha(k)) \end{bmatrix} = \sum_{i=1}^N \alpha_i(k) \begin{bmatrix} K_i & \hat{K}_i \end{bmatrix}. \quad (3.12)$$

Moreover, the augmented state vector includes the states of the actuator, leading to

$$\xi(k) = \begin{bmatrix} x^\top(k) & \hat{x}^\top(k) \end{bmatrix}^\top \in \mathbb{R}^{n+n_u}.$$

The resulting closed-loop system is represented by the block diagram in Figure 3.1, which is given by:

$$\begin{aligned} x(k+1) &= A(\alpha(k))x(k) + B(\alpha(k))\Phi(\mathbb{K}(\alpha(k))\xi(k)) + B_w(\alpha(k))\omega(k), \\ z(k) &= C_z(\alpha(k))x(k) + D_z(\alpha(k))\omega(k). \end{aligned} \quad (3.13)$$

Because the actuators have bounds on rate and magnitude, the resulting closed-loop system (3.13) presents a nonlinear behavior. Thus, global stability is generally unachievable, which is the case when $A(\alpha(k))$ is not exponentially stable. Consequently, a fundamental issue concerns the determination of the set of initial conditions $\mathcal{R}_a \subseteq \mathbb{R}^{n+n_u}$, called the region of attraction, from which the emanating trajectories, under $\omega = \mathbf{0}$, converge asymptotically to the origin. Moreover, assuming the disturbance signal $\omega \in \mathcal{W}$ with $\beta > 0$, it must be determined $\mathcal{R}_{\mathcal{E}_a} \subseteq \mathcal{R}_a$ from which the closed-loop trajectories do not leave the region of attraction ($\mathcal{R}_a$). Such a region can be non-convex and eventually not limited in certain directions (see (Tarbouriech et al., 2011, pp. 14)), yielding the challenge of determining an estimate of the region of attraction $\mathcal{R}_{\mathcal{E}_a} \subseteq \mathcal{R}_a$ as large as possible. Thus, to ensure regional stability in the presence of amplitude-bounded exogenous signals, the following definition, adapted from Sontag (2008), is considered.

Definition 3.1. *Consider a positive scalar $\beta > 0$ and any sequence $\omega \in \mathcal{W}$. The resulting closed-loop system is said to be regionally stable if for any initial state belonging to $\mathcal{R}_{\mathcal{E}_a}$ the resulting state trajectories remain bounded in $\mathcal{R}_{\mathcal{E}_a}$ for all $k \geq 0$. In this case, the set $\mathcal{R}_{\mathcal{E}_a}$ is positively invariant. Moreover, if the disturbance is null, and the state trajectories converge towards the origin, then the system is said input-to-state stable (ISS) and the set $\mathcal{R}_{\mathcal{E}_a}$ is contractive.*

The objective of this chapter is to search for less conservative convex conditions to design the control gains in (3.11), ensuring the local stability of nonlinear systems represented by polytopic discrete-time models under the presence of amplitude and rate saturating actuators and subject to amplitude disturbance signals satisfying (3.2). Therefore, two fundamental problems can be formulated as follows:

Problem 3.1. *Given the polytopic discrete-time system representation (3.7), the saturating actuators (3.3)–(3.4), the control law (3.11), and assuming no disturbances, i.e., $\omega = \mathbf{0}$, determine the state feedback control gains $\mathbb{K}(\alpha(k))$ and an estimate of the set of initial conditions $\mathcal{R}_{\mathcal{E}_a} \subseteq \mathcal{R}_a$ such that the resulting closed-loop system has its origin asymptotically stable, with trajectories converging towards the origin without leaving $\mathcal{R}_{\mathcal{E}_a}$.*

Problem 3.2. *Given the polytopic discrete-time system representation (3.7), the saturating actuators (3.3)–(3.4), the control law (3.11), and disturbance signals $\omega \in \mathcal{W}$ as in (3.2), determine the state feedback control gains $\mathbb{K}(\alpha(k))$ and an estimate of the set of initial conditions $\mathcal{R}_{\mathcal{E}_a} \subseteq \mathcal{R}_a$ such that the resulting closed-loop system has its origin stable and the trajectories do not leave $\mathcal{R}_{\mathcal{E}_a}$. Moreover, the designed controller must guarantee an upper limit γ to the induced ℓ_2/ℓ_∞ gain, in (3.5), between the disturbance signal $\omega(k)$ and the output $z(k)$, such that*

$$\|z(k)\|_2 = \sqrt{\gamma}(\|\omega(k)\|_2 + \mathfrak{N}), \quad (3.14)$$

where $\mathfrak{N}$ refers to the non-null initial conditions.

3.2 Preliminary Results

The dynamical equation (3.3) represents the model for the actuator block depicted in Figure 3.1, while the corresponding output is given by (3.4). To explore the (local) stability of the

system, it is essential to incorporate the actuator states into the system dynamics. The objective is to transform the closed-loop system into a Lur'e-type system, enabling the treatment of rate and magnitude saturation nonlinearities. This can be achieved by employing decentralized dead-zone functions, yielding the following representation.

$$\Psi_{\mathbf{M}}(k) = \text{sat}_{\mathbf{M}}(u(k)) - u(k)$$

and

$$\Psi_{\mathbf{R}}(k) = \text{sat}_{\mathbf{R}}(\Lambda \text{sat}_{\mathbf{M}}(u(k)) - \Lambda \hat{x}(k)) - (\Lambda \text{sat}_{\mathbf{M}}(u(k)) - \Lambda \hat{x}(k)).$$

Taking the control law as (3.11), these dead-zone functions are rewritten, respectively, as

$$\Psi_{\mathbf{M}}(k) = \text{sat}_{\mathbf{M}}(\mathbb{K}\xi(k)) - \mathbb{K}\xi(k)$$

and

$$\Psi_{\mathbf{R}}(k) = \text{sat}_{\mathbf{R}}\left((\hat{\Lambda} + \Lambda \mathbb{K}(\alpha(k)))\xi(k) + \Lambda \Psi_{\mathbf{M}}(k)\right) - \left((\hat{\Lambda} + \Lambda \mathbb{K}(\alpha(k)))\xi(k) + \Lambda \Psi_{\mathbf{M}}(k)\right),$$

with $\hat{\Lambda} = \begin{bmatrix} \mathbf{0} & -\Lambda \end{bmatrix}$. Finally, the resulting closed-loop system can be expressed in terms of the vector of augmented states as:

$$\begin{aligned} \xi(k+1) &= (\mathbb{A}(\alpha(k)) + \mathbb{B}_1 \mathbb{K}(\alpha(k)))\xi(k) + \mathbb{B}_1 \Psi_{\mathbf{M}}(k) + \mathbb{B}_2 \Psi_{\mathbf{R}}(k) + \mathbb{B}_w(\alpha(k))\omega(k), \\ z(k) &= \mathbb{C}_z(\alpha(k))\xi(k) + \mathbb{D}_z(\alpha(k))\omega(k) \end{aligned} \quad (3.15)$$

with

$$\begin{aligned} \mathbb{A}(\alpha(k)) &= \begin{bmatrix} A(\alpha(k)) & B(\alpha(k)) \\ \mathbf{0} & \mathbf{I} - \Lambda \end{bmatrix}, \quad \mathbb{B}_1 = \begin{bmatrix} \mathbf{0} \\ \Lambda \end{bmatrix}, \quad \mathbb{B}_2 = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}, \quad \mathbb{B}_w(\alpha(k)) = \begin{bmatrix} B_w(\alpha(k)) \\ \mathbf{0} \end{bmatrix}, \\ \mathbb{C}_z(\alpha(k)) &= \begin{bmatrix} C_z(\alpha(k)) & \mathbf{0} \end{bmatrix}, \quad \mathbb{D}_z(\alpha(k)) = \begin{bmatrix} D_z(\alpha(k)) & \mathbf{0} \end{bmatrix}. \end{aligned} \quad (3.16)$$

Two polyhedral sets, denoted $\mathbb{S}_{\mathbf{R}}$ and $\mathbb{S}_{\mathbf{M}}$, are introduced to capture the characteristics of rate and magnitude saturation, respectively. Notably, $\mathbb{S}_{\mathbf{R}}$ exhibits a nested saturating function as the rate saturation depends on the magnitude. The definitions of these sets are given in the sequence:

$$\mathbb{S}_{\mathbf{M}} = \left\{ \xi(k) \in \mathbb{R}^{n+n_u} : \left| (\mathbb{K}_{(\ell)}(\alpha(k)) - G_{\mathbf{M}(\ell)}(\alpha(k)))\xi(k) \right| \leq \mu_{\mathbf{M}(\ell)}, \ell = 1, \dots, n_u \right\}, \quad (3.17)$$

$$\begin{aligned} \mathbb{S}_{\mathbf{R}} = & \left\{ \xi(k) \in \mathbb{R}^{n+n_u}, \Psi_{\mathbf{M}}(k) \in \mathbb{R}^{n_u} : \right. \\ & \left. \left| \left(\begin{bmatrix} \hat{\Lambda}_{(\ell)} + \Lambda \mathbb{K}_{(\ell)}(\alpha(k)) & \Lambda_{(\ell)} \end{bmatrix} - G_{\mathbf{R}(\ell)}(\alpha(k)) \right) \begin{bmatrix} \xi(k) \\ \Psi_{\mathbf{M}}(k) \end{bmatrix} \right| \leq \mu_{\mathbf{R}(\ell)}, \ell = 1, \dots, n_u \right\}. \end{aligned} \quad (3.18)$$

$G_{\mathbf{M}}(\alpha(k)) \in \mathbb{R}^{n_u \times (n+n_u)}$ and $G_{\mathbf{R}}(\alpha(k)) = \begin{bmatrix} G_{\mathbf{R}_1}(\alpha(k)) & G_{\mathbf{R}_2}(\alpha(k)) \end{bmatrix}$ are time-varying parameter-dependent matrices, where $G_{\mathbf{R}_1}(\alpha(k)) \in \mathbb{R}^{n_u \times (n+n_u)}$ and $G_{\mathbf{R}_2}(\alpha(k)) \in \mathbb{R}^{n_u \times n_u}$.

By employing the sets $\mathbb{S}_{\mathbf{M}}$ and $\mathbb{S}_{\mathbf{R}}$ in conjunction with the generalized sector condition (Gomes da Silva Jr. and Tarbouriech, 2005), the control gain is designed to accommodate a certain level of saturation in the control signal $u(k)$. The lemma in the sequence is leveraged to address the challenges posed by saturating actuators. The conditions in this lemma were used in Oliveira et al. (2022) as an adaptation of Tarbouriech et al. (2006, Lemma 1) by considering parameter dependent matrices $G_{\mathbf{M}}(\alpha(k))$, $G_{\mathbf{R}_1}(\alpha(k))$, $G_{\mathbf{R}_2}(\alpha(k)) \in \mathbb{R}^{n_u \times (n+n_u)}$.

Lemma 3.1. *If $\xi(k) \in \mathbb{S}_M$, $\begin{bmatrix} \xi^\top(k) & \Psi_M^\top(k) \end{bmatrix}^\top \in \mathbb{S}_R$ then the nonlinearities $\Psi_M(k)$ and $\Psi_R(k)$ satisfy the following inequalities:*

$$\Psi_M^\top(k) T_M (\Psi_M(k) + G_M(\alpha(k))\xi(k)) \leq 0, \quad (3.19)$$

$$\Psi_R^\top(k) T_R \left(\Psi_R(k) + G_R(\alpha(k)) \begin{bmatrix} \xi(k) \\ \Psi_M(k) \end{bmatrix} \right) \leq 0, \quad (3.20)$$

for any diagonal matrices $\mathbf{0} < T_M$ and $\mathbf{0} < T_R$ belonging to $\mathbb{R}^{n_u \times n_u}$.

Proof: It follows the same developments done in [Gomes da Silva Jr. and Tarbouriech \(2005\)](#); [Tarbouriech et al. \(2011\)](#). First, regarding (3.19), assuming that $u(k)$ and $G_M(\alpha(k))\xi(k)$ belong to $\mathbb{S}_M$, it follows that $\mu_{M(\ell)} - u_{(\ell)}(k) + G_{M(\ell)}(\alpha(k))\xi(k) \geq 0$ and $-\mu_{M(\ell)} - u_{(\ell)}(k) + G_{M(\ell)}(\alpha(k))\xi(k) \leq 0$, $\ell = 1, \dots, n_u$. Consequently, three cases must be analyzed: *Case 1:* if $u_{(\ell)}(k) > \mu_{M(\ell)}$, hence $\Psi_{M(\ell)}(k) = \mu_{M(\ell)} - u_{(\ell)}(k) < 0$ and

$$\Psi_{M(\ell)}^\top(k) T_{M(\ell\ell)} (\Psi_{M(\ell)}(k) + G_{M(\ell)}(\alpha(k))\xi(k)) = \Psi_{M(\ell)}^\top(k) T_{M(\ell\ell)} (\mu_{M(\ell)} - u_{(\ell)}(k) + G_{M(\ell)}(\alpha(k))\xi(k)) \leq 0$$

with $T_{M(\ell\ell)} > 0$; *Case 2:* if $-\mu_{M(\ell)} \leq u_{(\ell)}(k) \leq \mu_{M(\ell)}$, hence $\Psi_{M(\ell)}(k) = 0$ and

$$\Psi_{M(\ell)}^\top(k) T_{M(\ell\ell)} (\Psi_{M(\ell)}(k) + G_{M(\ell)}(\alpha(k))\xi(k)) = 0,$$

$\forall T_{M(\ell\ell)} > 0$; *Case 3:* otherwise, $u_{(\ell)}(k) < -\mu_{M(\ell)}$, hence $\Psi_{M(\ell)}(k) = -\mu_{M(\ell)} - u_{(\ell)}(k) > 0$ and

$$\begin{aligned} & \Psi_{M(\ell)}^\top(k) T_{M(\ell\ell)} (\Psi_{M(\ell)}(k) + G_{M(\ell)}(\alpha(k))\xi(k)) \\ &= \Psi_{M(\ell)}^\top(k) T_{M(\ell\ell)} (-\mu_{M(\ell)} - u_{(\ell)}(k) + G_{M(\ell)}(\alpha(k))\xi(k)) \leq 0, \end{aligned}$$

with $T_{M(\ell\ell)} > 0$. Consequently, since $u(k)$ and $G_M(\alpha(k))\xi(k)$ are elements of $\mathbb{S}_M$, the following inequality holds:

$$\Psi_M^\top(k) T_M (\Psi_M(k) + G_M(\alpha(k))\xi(k)) \leq 0,$$

for all $T_{M(\ell\ell)} > 0$, $\ell = 1, \dots, n_u$, verifying (3.19).

Similarly, the same steps are done with (3.20) for $\mathbb{S}_R$, changing $u(k)$ by $\Lambda \text{sat}_M(u(k)) - \Lambda \hat{x}(k)$, $\Psi_M(k)$ by $\Psi_R(k)$, $G_M(\alpha(k))\xi(k)$ by $G_R(\alpha(k)) \begin{bmatrix} \xi^\top(k) & \Psi_M^\top(k) \end{bmatrix}^\top$, and μ_R by μ_M . This concludes the proof. $\blacksquare$

It is worth noting that a key outcome of Lemma 3.1 is the existence of signals $G_M(\alpha(k))\xi(k)$ and $G_{R_1}(\alpha(k))\xi(k) + G_{R_2}(\alpha(k))\Psi_M(k)$ which compensate the control signal $u(k)$ in such a way that they belong to the sets $\mathbb{S}_M$ and $\mathbb{S}_R$, thereby ensuring that the conditions are satisfied.

Hereafter, $\mathcal{L}_V(\eta)$ denotes the level set corresponding to a Lyapunov function $V(\xi(k))$, where $\xi(k) \in \mathbb{R}^n$ is the state, parameter $\alpha(k) \in \mathbb{R}^N$ belongs to the convex hull Γ_N given in (3.8), and η is a positive scalar. Consequently,

$$\mathcal{L}_V(\eta) = \left\{ \xi(k) \in \mathbb{R}^n : V(\xi(k)) \leq \eta^{-1}, \alpha(k) \in \Omega \right\}.$$

3.3 Main Results

This section presents convex conditions allowing the design of state feedback gains that provide solutions to Problems 3.1 and 3.2. The conditions are formulated in terms of LMIs, which can be solved efficiently by numerical interior point methods freely available. The proposed solution explores the polytopic structure of the model, offering an equally polytopic-based control gain to stabilize the closed-loop system locally.

Theorem 3.1. *Consider the polytopic discrete-time system under rate and magnitude saturating actuators given by (3.7)-(3.11), matrices $\mathbb{A}_i$, $\mathbb{B}_1$, $\mathbb{B}_2$, $\mathbb{B}_{wi}$, $\mathbb{C}_{zi}$, and $\mathbb{D}_{zi}$ obtained as in (3.16), and the known symmetric saturation limits μ_M and μ_R for magnitude and rate, respectively. Assume that there exist positive scalars η , β , τ_2 , and $0 < \tau_1 < 1$, positive definite and symmetric matrices $\tilde{P}_i \in \mathbb{R}^{(n+n_u) \times (n+n_u)}$, diagonal positive matrices S_M , $S_R \in \mathbb{R}^{n_u \times n_u}$, matrices X_i , W_{Mi} , $W_{R1i} \in \mathbb{R}^{n_u \times (n+n_u)}$, $W_{R2i} \in \mathbb{R}^{n_u \times n_u}$, $\tilde{U} \in \mathbb{R}^{(n+n_u) \times (n+n_u)}$, for $i, j = 1, \dots, N$, such that the following inequalities:*

$$\begin{bmatrix} -\tau_1 \tilde{P}_i & -\tilde{U}^\top \mathbb{A}_i^\top - X_i^\top \mathbb{B}_1^\top & -W_{Mi}^\top & -W_{R1i}^\top & \mathbf{0} & \tilde{U}^\top \mathbb{C}_{zi}^\top \\ \star & \tilde{P}_j + \tilde{U} + \tilde{U}^\top & -\mathbb{B}_1 S_M & -\mathbb{B}_2 S_R & -\mathbb{B}_{wi} & \mathbf{0} \\ \star & \star & -2S_M & -W_{R2i}^\top & \mathbf{0} & \mathbf{0} \\ \star & \star & \star & -2S_R & \mathbf{0} & \mathbf{0} \\ \star & \star & \star & \star & -\tau_2 \mathbf{I} & \mathbb{D}_{zi}^\top \\ \star & \star & \star & \star & \star & -\gamma \mathbf{I} \end{bmatrix} < \mathbf{0}, \quad (3.21)$$

$$\begin{bmatrix} -\tilde{P}_i & X_{i(\ell)}^\top - W_{Mi(\ell)}^\top \\ \star & -\eta \mu_{M(\ell)}^2 \end{bmatrix} \leq \mathbf{0}, \quad (3.22)$$

$$\begin{bmatrix} -\tilde{P}_i & W_{Mi}^\top & \tilde{U}^\top \hat{\Lambda}_{(\ell)}^\top + (\Lambda X_i)_{(\ell)}^\top - W_{R1i(\ell)}^\top \\ \star & -2S_M & S_M \Lambda_{(\ell)}^\top - W_{R2i(\ell)}^\top \\ \star & \star & -\eta \mu_{R(\ell)}^2 \end{bmatrix} \leq \mathbf{0}, \quad (3.23)$$

$$-(1 - \tau_1)\beta + \tau_2\eta < 0, \quad (3.24)$$

and

$$\begin{bmatrix} -\tilde{P}_i & \tilde{U}^\top L_{(r)}^\top \\ \star & -\eta \rho_{(r)}^2 \end{bmatrix} \leq \mathbf{0}, \quad (3.25)$$

are verified $\forall \ell = 1, \dots, n_u$ and $\forall r = 1, \dots, r_c$. Then, the control law given by

$$u(k) = K(\alpha(k))x(k) + \hat{K}(\alpha(k))\hat{x}(k) = \mathbb{K}(\alpha(k))\xi(k),$$

with control gain matrices computed as

$$\mathbb{K}(\alpha(k)) = \sum_{i=1}^N \alpha_i(k) X_i \tilde{U}^{-1}, \quad \alpha \in \Gamma_N \quad (3.26)$$

guarantees that

1. the region $\mathcal{R}_{\mathcal{E}_a} = \mathcal{L}_V(\eta) \subseteq \mathcal{R}_a$ is an estimate of the domain of attraction for the origin for every initial condition belonging to it whenever $\omega = \mathbf{0}$;

2. the trajectories of the closed-loop system do not leave the set $\mathcal{R}_{\mathcal{E}_a}$ for every initial state inside it, for all $k \geq 0$ and $\alpha(k) \in \Gamma_N$, for $\omega \neq \mathbf{0}$ with $\omega \in \mathcal{W}$;
3. an upper limit of the induced ℓ_2/ℓ_∞ gain such that $\|z(k)\|_2 \leq \sqrt{\gamma}(\|\omega(k)\|_2 + V(\xi(0)))$ $\forall k \rightarrow \infty$ is given by $\sqrt{\gamma}$.

The estimate of the region of attraction $\mathcal{R}_{\mathcal{E}_a}$ can be obtained by the level set $\mathcal{L}_V(\eta)$ defined as

$$\mathcal{L}_V(\eta) = \bigcap_{\alpha(k) \in \Gamma_N} \mathcal{E}(P(\alpha(k)), \eta) = \bigcap_{i \in 1, \dots, N} \mathcal{E}(P_i, \eta). \quad (3.27)$$

Proof: The local stability can be investigated considering the following parameter-dependent Lyapunov function candidate:

$$V(\xi(k)) = \xi^\top(k) P(\alpha(k)) \xi(k), \quad (3.28)$$

with

$$P(\alpha(k)) = \sum_{i=1}^N \alpha_i(k) P_i > \mathbf{0}, \quad (3.29)$$

and $\alpha(k) \in \Gamma_N$ must satisfy the sequence set Ω given in (3.9). Hence, the conditions (3.21)-(3.25) can ensure that

$$\begin{aligned} \kappa_1(\xi(k)) &\leq V(\xi(k)) \leq \kappa_2(\xi(k)), \\ \Delta V(\xi(k)) &= V(\xi(k+1)) - V(\xi(k)) \leq -\kappa_3(\xi(k)), \end{aligned} \quad (3.30)$$

are satisfied for $\xi(k) \in \mathcal{R}_{\mathcal{E}_a} \subseteq \mathbb{R}^{n+n_u}$, with class- $\mathcal{K}$ functions $\kappa_i(\xi(k))$, $i \in \{1, 2, 3\}$, verified for the sequences of $\alpha(k) \in \Omega$. Such functions are given in the sequence of this proof. Thus, assuming that the inequality (3.21) is feasible, the positive definite matrices are obtained: $\mathbf{0} < \tilde{P}_i$, $\mathbf{0} < S_M$, and $\mathbf{0} < S_R$. Moreover, $\tilde{U}$ is a full-rank matrix due to the negativity of the block (2, 2) in (3.21). The local stability of the closed-loop system can be ensured as follows: First, replace $\tilde{P}_i$, X_i , S_M , S_R , W_{M_i} , $W_{R_1 i}$, and $W_{R_2 i}$, by $\tilde{U}^\top P_i \tilde{U}$, $\mathbb{K}_i \tilde{U}$, T_M^{-1} , T_R^{-1} , $G_{M_i} \tilde{U}$, $G_{R_1 i} \tilde{U}$, and $G_{R_2 i} \tilde{U}$, respectively. Next, multiply the result by $\alpha_i(k)$, $\alpha(k+1)_{(j)}$, $\alpha(k) \in \Gamma_N$ and sum it up for $i, j = 1, \dots, N$ to get an inequality depending on the weighting function at the instants k and $k+1$. Thanks to the regularity of $\tilde{U}$, one can apply a similarity transformation by pre-multiplying the resulting inequality by $\Upsilon^\top = \text{diag}\{\tilde{U}^{-\top}, \tilde{U}^{-\top}, T_M, T_R, \mathbf{I}, \mathbf{I}\}$ and post-multiplying it by Υ . This can be seen as an application of the Finsler Lemma (de Oliveira and Skelton, 2001). In the sequence, apply the Schur complement to reduce the dimensions of the inequality. Subsequently, pre-multiply the resulting inequality by $\nu^\top$ and post-multiply it by ν , with $\nu^\top = \left[\xi^\top(k) \quad \xi^\top(k+1) \quad \Psi_M^\top(k) \quad \Psi_R^\top(k) \quad \omega^\top(k) \right]^\top$. The resulting expression depends on the augmented states of the system and can be related to the Lyapunov candidate function by denoting $\Delta V(\xi(k)) = \xi^\top(k+1) P(\alpha(k+1)) \xi(k+1) - \xi^\top(k) P(\alpha(k)) \xi(k)$, and from (3.15) replacing $\mathbb{C}_z(\alpha(k)) \xi(k) + \mathbb{D}_z(\alpha(k)) \omega(k)$ by $z(k)$, which yields

$$\begin{aligned} \Delta V(\xi(k)) &+ (1 - \tau_1)(\xi^\top(k) P(\alpha(k)) \xi(k) - \eta^{-1}) - 2\Psi_M^\top(k) T_M \Psi_M(k) - 2\Psi_M^\top(k) T_M G_M(\alpha(k)) \xi(k) \\ &- 2\Psi_R^\top(k) T_R \Psi_R(k) - 2\Psi_R^\top(k) T_R G_{R_1}(\alpha(k)) \xi(k) - 2\Psi_R^\top(k) T_R G_{R_2}(\alpha(k)) \Psi_M(k) \\ &+ \tau_2(\beta^{-1} - \omega^\top(k) \omega(k)) + \gamma^{-1} z^\top(k) z(k) \leq 0. \end{aligned} \quad (3.31)$$

where τ_1 and τ_2 come as multipliers from the S-procedure.

Thus, with (3.21) verified and assuming that $\xi(k) \in \mathbb{S}_M$ and $\begin{bmatrix} \xi^\top(k) & \Psi_M^\top(k) \end{bmatrix}^\top \in \mathbb{S}_R$, the conditions of Lemma 3.1 are fulfilled (which is demonstrated in the sequel) and then (3.31) can be used to compute $\kappa_1(\xi(k)) = h_1 \|\xi(k)\|$, with $h_1 = \min_{i=1, \dots, N} \text{eig}(\tilde{U}^{-\top} \tilde{P}_i \tilde{U}^{-1})$, $\kappa_2(\xi(k)) = h_2 \|\xi(k)\|$, with $h_2 = \max_{i=1, \dots, N} \text{eig}(\tilde{U}^{-\top} \tilde{P}_i \tilde{U}^{-1})$. Additionally, with $\kappa_3(\xi(k)) = h_3 \|\xi(k)\|$, with a small enough $h_3 > 0$, one gets that equation (3.30) is verified, thus ensuring that $V(\xi(k))$ is a Lyapunov function. Consequently, under the control law (3.11), the state trajectories of the closed-loop system (3.13) remain bounded for any amplitude-bounded disturbance signal that satisfies (3.2). Furthermore, equation (3.31) also ensures that there is an upper bound on the ℓ_2/ℓ_∞ gain between the disturbance and the controlled output, and the equation (3.14) is verified under null initial conditions. Thus, the closed-loop performance is guaranteed for the ℓ_2/ℓ_∞ gain for all admissible $\alpha(k) \in \Gamma_N$.

Assuming the feasibility of (3.22), the inclusion of the contractive set $\mathcal{L}_V(\eta)$ in the region defined by $\mathbb{S}_M \cap \mathbb{S}_R$ can be demonstrated as follows: First, $\tilde{P}_i$, X_i , and W_{Mi} are replaced by $\tilde{U}^\top P_i \tilde{U}$, $\mathbb{K}_i \tilde{U}$, and $G_{Mi} \tilde{U}$, respectively, in (3.22). Subsequently, the resulting inequality is multiplied by $\alpha_i(k)$, $\alpha(k) \in \Gamma_N$, and summed over $i = 1, \dots, N$. Then, the parameter-dependent inequality is pre- and post-multiplied by $\Upsilon = \text{diag}\{\tilde{U}^{-\top}, 1\}$ and its transpose to apply Schur's complement. By pre- and post-multiplying it by $\xi^\top(k)$ and $\xi(k)$, it follows that:

$$\xi^\top(k) (\mathbb{K}_{(\ell)}(\alpha(k)) - G_{M(\ell)}(\alpha(k)))^\top (\eta \mu_{M(\ell)}^{-2}) (\mathbb{K}_{(\ell)}(\alpha(k)) - G_{M(\ell)}(\alpha(k))) \xi(k) - \xi^\top(k) P(\alpha(k)) \xi(k) \leq 0,$$

$\forall \ell = 1, \dots, n_u$, which implies that $\mathcal{E}(P(\alpha(k)), \eta) \subset \mathbb{S}_M$, as in (3.17). Then, to guarantee the inclusion in $\mathbb{S}_R$, the feasibility of (3.23) is assumed, and similar steps are followed: $\tilde{P}_i$, X_i , S_M , W_{Mi} , $W_{R_1 i}$, and $W_{R_2 i}$ are replaced by $\tilde{U}^\top P_i \tilde{U}$, $\mathbb{K}_i \tilde{U}$, T_M^{-1} , $G_{Mi} \tilde{U}$, $G_{R_1 i} \tilde{U}$, and $G_{R_2 i} S_M$ in (3.23), respectively. The resulting inequality is multiplied by $\alpha_i(k)$, $\alpha(k) \in \Gamma_N$ and summed over $i = 1, \dots, N$ to get a parameter-dependent inequality that, thanks to the convexity, is satisfied for all $\alpha(k) \in \Gamma_N$. Next, the congruence transformation is applied by pre- and post-multiplying it by $\text{diag}\{\tilde{U}^{-\top}, T_M, 1\}$ and its transpose. With Schur's complement and pre- and post-multiply the result by $\zeta^\top = \begin{bmatrix} \xi^\top(k) & \Psi_M^\top(k) \end{bmatrix}$ and ζ , it follows that

$$\zeta^\top \left(\Theta^\top (\eta \mu_{R(\ell)}^{-2}) \Theta - \begin{bmatrix} P(\alpha(k)) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \right) \zeta \leq 0, \forall \ell = 1, \dots, n_u,$$

with $\Theta = \begin{bmatrix} \hat{\Lambda}_{(\ell)} + \Lambda \mathbb{K}_{(\ell)}(\alpha(k)) - G_{R_1(\ell)} & \Lambda_{(\ell)} - G_{R_2(\ell)} \end{bmatrix}$, $\ell = 1, \dots, n_u$. Therefore, it can be concluded that if $\xi(0)$ belongs to $\mathcal{R}_{\mathcal{E}_a} \equiv \mathcal{L}_V(\eta)$, then it is contained in $\mathbb{S}_M \cap \mathbb{S}_R$ and remains inside such a set for all disturbance signals belonging to (3.2).

Lastly, assuming that (3.25) is verified, $\tilde{P}_i$ is replaced by $\tilde{U}^\top P_i \tilde{U}$, the resulting inequality is then multiplied by $\alpha_i(k)$ and summed over $i = 1, \dots, N$. With $\text{diag}\{\tilde{U}^{-\top}, 1\}$ as a congruence transformation matrix, applying Schur's complement, and pre- and post-multiplying the resulting inequality by $\xi^\top(k)$ and $\xi(k)$, it follows that

$$\xi^\top(k) L_{(r)}^\top (\eta \rho_{(r)}^{-2}) L_{(r)} \xi(k) - \xi^\top(k) P(\alpha(k)) \xi(k) \leq 0, \quad \forall r = 1, \dots, r_c.$$

Therefore, it can be concluded that $\mathcal{L}_V(\eta) \subseteq \mathbb{S}_x$, ensuring the local stability in the validity domain of the polytopic model. Consequently, $\mathcal{L}_V(\eta) \subseteq (\mathbb{S}_x \cap \mathbb{S}_M \cap \mathbb{S}_R)$, thereby concluding the proof. $\blacksquare$

Remark 3.2. The estimate of the region of attraction $\mathcal{L}_V(\eta)$ can be seen as a particular condition of (Figueiredo et al., 2021, Lemma 2, with $g = 1$) that recovers Jungers and Castelan (2011) for a parameter-dependent Lyapunov function. By this lemma, the estimate of the region of attraction $\mathcal{R}_{\mathcal{E}_a}$ is computed through finite-dimensional conditions using the intersection of the level sets of P_i , $\forall i = 1, \dots, N$. Otherwise, in the quasi-LPV case, the time-varying parameters $\alpha(k)$ should be replaced by their expressions depending on $x(k)$ into the constraints

$$\xi^\top(k)P(\alpha(k))\xi(k) \leq \eta^{-1} = \mathcal{E}(P(\alpha(k)), \eta), \quad (3.32)$$

to obtain a less conservative estimate $\mathcal{R}_{\mathcal{E}_a}$. For details, please see (Leite et al., 2021, pp. 373, Example 1).

Remark 3.3. It is important to highlight the consequence of variable τ_1 , which governs the contractivity rate of the Lyapunov function. Incorporating the performance specification into the closed-loop system ensures the inequality $\Delta V(\xi(k)) + (1 - \tau_1)V(\xi(k)) \leq 0$, which is further simplified to $V(\xi(k+1)) - \tau_1 V(\xi(k)) \leq 0$ with $\tau_1 \in (0, 1]$. This criterion implies that a smaller value of τ_1 leads to a faster decay of the system states toward equilibrium.

When considering null input disturbance, i.e., for $\omega = \mathbf{0}$, the following particular case is obtained, recovering the conditions proposed in Oliveira et al. (2020):

Corollary 3.1. If there exists positive definite and symmetric matrices $\tilde{P}_i \in \mathbb{R}^{(n+n_u) \times (n+n_u)}$, diagonal positive matrices $S_M, S_R \in \mathbb{R}^{n_u \times n_u}$, matrices $X_i, W_{M_i}, W_{R_1 i} \in \mathbb{R}^{n_u \times (n+n_u)}$, $W_{R_2 i} \in \mathbb{R}^{n_u \times n_u}$, and $\tilde{U} \in \mathbb{R}^{(n+n_u) \times (n+n_u)}$ such that inequalities (3.22), (3.23), (3.25), and

$$\begin{bmatrix} -\tilde{P}_i & -\tilde{H}^\top \mathbb{A}_i^\top - X_i^\top \mathbb{B}_1^\top & -W_{M_i}^\top & -W_{R_1 i}^\top \\ \star & \tilde{P}_j + \tilde{U} + \tilde{U}^\top & -\mathbb{B}_1 S_M & -\mathbb{B}_2 S_R \\ \star & \star & -2S_M & -W_{R_2 i}^\top \\ \star & \star & \star & -2S_R \end{bmatrix} < \mathbf{0}, \quad (3.33)$$

are verified for $i, j = 1, \dots, N$, $\ell = 1, \dots, n_u$, and $r = 1, \dots, r_c$. Then, the control gain matrices given by (3.26) yield the control law (3.11) which guarantees asymptotic stability in the closed-loop system and an estimate of the region of attraction given by $\mathcal{R}_{\mathcal{E}_a} = \mathcal{L}_V(\eta)$.

Remark 3.4. It is worth mentioning that Theorem 3.1 and Corollary 3.1 conditions can be cast in the robust control framework whenever the parameter $\alpha(k)$ is unavailable online. Thus, it is enough to get a robust version of these conditions with $X_i = X$, $i = 1, \dots, N$ in both conditions. In such a case the control law (3.11) can be simplified as

$$u(k) = Kx(k) + \hat{K}\hat{x}(k) = \mathbb{K}\xi(k),$$

where the gain $\mathbb{K}$ (or equivalently K and $\hat{K}$) correspond to the robust version of that in (3.26) and can be computed as

$$\mathbb{K} = X\tilde{U}^{-1}.$$

3.3.1 Optimization Procedures

Based on Theorem 3.1, sufficient solutions for Problems 3.1 and 3.2 can be obtained. However, it is possible to design controllers that optimize specific objectives, such as the size of the estimate of the region of attraction, the allowable disturbance amplitude, or the bounds of the disturbance rejection. The following convex optimization procedures are presented to achieve these objectives.

Maximization of $\mathcal{R}_{\mathcal{E}_a}$: Assuming a maximum tolerable amplitude bound for the disturbance signal, β^{-1} , a control law can be found that maximizes the region of attraction estimate, $\mathcal{R}_{\mathcal{E}_a}$, while ensuring the corresponding closed-loop trajectories initiated within it are bounded. Considering the ellipsoidal set $\mathcal{E}(R, \eta) \subseteq \mathcal{L}_V(\eta)$, ensuring to $R \geq P_i$ (Seuret and Gomes Da Silva Jr., 2012), with a positive definite matrix $R \in \mathbb{R}^n$ and

$$\begin{bmatrix} -R & \mathbf{I} \\ \star & (\tilde{P}_i + \tilde{U} + \tilde{U}^\top) \end{bmatrix} \geq \mathbf{0}. \quad (3.34)$$

Hence, the size of $\mathcal{R}_{\mathcal{E}_a}$ can be enlarged using the following convex optimization approach:

$$\mathcal{J}_1 : \begin{cases} \min & \text{trace}(R) \\ \text{subject to:} & \text{LMIs (3.21)–(3.25), and (3.34),} \end{cases} \quad (3.35)$$

$\forall i, j = 1, \dots, N, \ell = 1, \dots, n_u$, and $r = 1, \dots, r_c$.

Remark 3.5. The results obtained in $\mathcal{J}_1$ can be further improved by applying the relaxation algorithm described in Silva et al. (2020), inspired by the Frank–Wolfe algorithm. In this setting, the procedure aims at enlarging the estimated region of attraction computed through (3.27) by iteratively maximizing R .

Starting from a feasible solution, the first step ensures the feasibility of the optimization problem, and the feasibility of the subsequent iterations is preserved by construction. At each iteration, the weights associated with the optimization directions in the objective function are updated, prioritizing the most restrictive directions that limit the growth of R . This strategy leads to a progressive enlargement of the region characterized by R .

Maximization of the allowable disturbance: Another possibility is to maximize the amplitude of the allowed disturbance while maintaining system stability. In this case, the goal is to design the control gains that maximize the amplitude of admissible perturbations $\omega(k) \in \mathcal{W}$, ensuring that the trajectories inside $\mathcal{R}_{\mathcal{E}_a}$ do not leave it. For this purpose, the following convex optimization procedure can be applied:

$$\mathcal{J}_2 : \begin{cases} \min & \beta \\ \text{subject to:} & \text{LMIs (3.21)–(3.25), and (3.34),} \end{cases} \quad (3.36)$$

$\forall i, j = 1, \dots, N, \ell = 1, \dots, n_u$, and $r = 1, \dots, r_c$.

Minimization of a bound to the ℓ_2/ℓ_∞ gain: Lastly, the induced ℓ_2/ℓ_∞ gain between the disturbance signal and the regulated or performance output can be minimized, which corresponds

to maximizing the disturbance rejection for a given amplitude disturbance bound. Therefore, bound to the ℓ_2/ℓ_∞ gain can be minimized by the following convex optimization procedure:

$$\mathcal{J}_3 : \begin{cases} \min & \gamma \\ \text{subject to:} & \text{LMIs (3.21)–(3.25), and (3.34)} \end{cases} \quad (3.37)$$

$\forall i, j = 1, \dots, N, \ell = 1, \dots, n_u$, and $r = 1, \dots, r_c$.

3.4 Illustrative Examples

Three examples are presented to illustrate the effectiveness of the proposed conditions for controller design. The first example explores the estimated region of attraction of a polytopic LPV model and investigates how this region changes when the model is interpreted as a T-S fuzzy system. Notably, this example demonstrates that the proposed approach aligns with the conditions required to apply Algorithm 1 in [Silva et al. \(2020\)](#), enabling larger estimates of the region of attraction. The second example addresses the tracking problem in a polytopic LPV model where the exogenous signal serves as a reference to be followed. In this case, the ℓ_2/ℓ_∞ gain is analyzed, showing that the proposed method yields a closed-loop system with improved performance compared to existing literature results. Finally, the section concludes with real-time experiments conducted on a nonlinear coupled tank system, validating the efficacy of the proposed method and feasibility for practical implementation.

3.4.1 Estimated Region of Attraction of LPV and T-S Fuzzy Models

Consider a T-S fuzzy model, borrowed from [Guerra and Vermeiren \(2004\)](#), subject to control inputs limited in rate and magnitude as described by (3.15) with matrices

$$A_1 = \begin{bmatrix} 1 & -\sigma \\ -1 & -0.5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & \sigma \\ -1 & -0.5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 5 + \sigma \\ 2\sigma \end{bmatrix}, \quad B_2 = \begin{bmatrix} 5 - \sigma \\ -2\sigma \end{bmatrix}, \quad (3.38)$$

and $\sigma > 0$. The bounds for rate and magnitude saturation, given by $\mu_M = 0.4$ and $\mu_R = 0.2$ respectively, are symmetric as discussed in the actuator modeling, Section 3.1. Additionally, the presence of an ideal rate limiter with $\Lambda = 1$ is assumed. The proposed model is valid for $|x_1(k)| \leq \sigma$. According to Remark 3.1, the region $\mathbb{S}_x$ can be characterized as stated in (3.6), with $L = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ and $\rho = \sigma$. Assuming null input disturbance ($\omega = 0$), the size of the region of attraction $\mathcal{R}_{\mathcal{E}_a}$ is evaluated. Due to the characteristics of the model, it represents a typical LPV system, enabling the weighting functions $\alpha_i(k)$, $i = 1, 2$, to assume arbitrary sequence values within Ω , as long as $\alpha(k) \in \Gamma_N$ in (3.8). Defining $\sigma = 0.3$, by utilizing Corollary 3.1 (which recovers the conditions in [Oliveira et al. \(2022\)](#)) along with the optimization procedure $\mathcal{J}_1$ given in (3.35), the controller gains are obtained as:

$$\begin{bmatrix} K_1 & | & K_2 \end{bmatrix} = \begin{bmatrix} -0.1872 & 0.0435 & -0.8981 & | & -0.1887 & -0.0624 & -0.8130 \end{bmatrix}, \quad (3.39)$$

and matrices associated with the parameter-dependent Lyapunov function:

$$\left[\begin{array}{c|c} P_1 & P_2 \end{array} \right] = \left[\begin{array}{ccc|ccc} 20.2347 & -2.5778 & 55.3227 & 21.6573 & 2.2637 & 54.9110 \\ -2.5778 & 4.5102 & -20.1163 & 2.2637 & 3.8310 & 8.3013 \\ 55.3227 & -20.1163 & 345.7926 & 54.9110 & 8.3013 & 293.4345 \end{array} \right]. \quad (3.40)$$

By using (3.40), an estimate for the region of attraction $\mathcal{R}_{\mathcal{E}_a} = \mathcal{L}_{\mathcal{V}}(1)$ can be obtained. Such an estimate involves the intersection of ellipses characterized by P_i , where $i = 1, 2$, as outlined in (3.27). However, it is possible to further expand this region by employing an algorithm inspired by Frank-Wolfe methods (Silva et al., 2020, Algorithm 1). Algorithm 1 from Silva et al. (2020) was run for eight iterations, resulting in the difference between the objective function values of the last two iterations falling below 10^{-4} . Consequently, the following controller gain is obtained:

$$\left[\begin{array}{c|c} K_1 & K_2 \end{array} \right] = \left[\begin{array}{ccc|ccc} -0.1905 & 0.0521 & -0.9722 & -0.1905 & -0.0584 & -0.8694 \end{array} \right], \quad (3.41)$$

and the Lyapunov matrices:

$$\left[\begin{array}{c|c} P_1 & P_2 \end{array} \right] = \left[\begin{array}{ccc|ccc} 19.8947 & -3.1811 & 53.2379 & 21.8870 & 3.2360 & 51.5366 \\ -3.1811 & 1.7724 & -20.3602 & 3.2360 & 1.3520 & 14.8350 \\ 53.2379 & -20.3602 & 324.5559 & 51.5366 & 14.8350 & 247.5603 \end{array} \right]. \quad (3.42)$$

Figure 3.2 shows a cross-section of the estimated region of attraction $\mathcal{R}_{\mathcal{E}_a}$ at $\hat{x} = \mathbf{0}$ (state of the actuator) in the phase plane for two cases: on the left, the cross-section obtained after applying the optimization procedure $\mathcal{J}_1$; on the right, the region achieved after eight iterations of (Silva et al., 2020, Algorithm 1), as implemented in this approach. The area of the intersection of regions related to (3.40) (solid black line, left plot) is equal to 0.2889, while its counterpart on the right-hand side of Figure 3.2 is 0.3534, representing an increase of 22.31%.

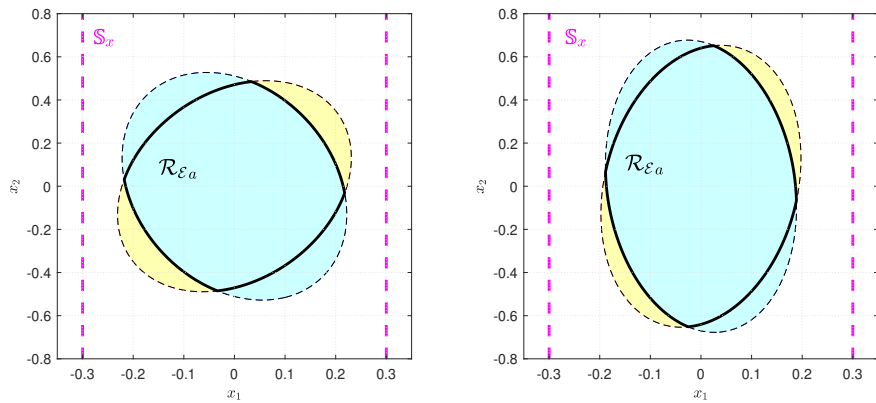


Figure 3.2: Cut of the estimate region of attraction $\mathcal{R}_{\mathcal{E}_a}$ at $\hat{x} = \mathbf{0}$ obtained with optimization procedure $\mathcal{J}_1$ given in (3.35) (left) and the enhanced result achieved with the use of (Silva et al., 2020, Algorithm 1) (right). The intersection area has been augmented by 22.31% in the right plot.

Table 3.1 allows for the analysis of area enlargement with respect to the σ value (column 1). The results are obtained using the optimization procedure $\mathcal{J}_1$ (column 2) and the application of (Silva et al., 2020, Algorithm 1), employing the same stopping criteria as before: a reduction

of the objective function by less than 10^{-4} . It can be observed that, as the σ value decreases, resulting in the convergence of polytope vertices, the effectiveness of applying (Silva et al., 2020, Algorithm 1) becomes increasingly apparent.

Table 3.1: Comparison of the estimated (intersection) areas achieved with optimization procedure $\mathcal{J}_1$ and with the application of (Silva et al., 2020, Algorithm 1) for different values of σ .

σ	$\mathcal{J}_1$ Area $\mathcal{R}_{\mathcal{E}_a}$	$\mathcal{J}_1$ with (Silva et al., 2020, Algorithm 1) Area $\mathcal{R}_{\mathcal{E}_a}$	Area augmentation (%)
0.999	0.7373	0.7535	2.68
0.5	0.5990	0.6237	4.13
0.3	0.2889	0.3534	22.31
0.1	0.0635	0.1153	81.59

Therefore, using (Silva et al., 2020, Algorithm 1) shows a simple and worthwhile tool to maximize the estimate of the region of attraction for the present proposal.

Remark 3.6. *The presented example evaluates the feasibility of the proposed conditions for $\sigma < 1$. In Guerra and Vermeiren (2004), an alternative approach is presented, which relaxes the feasibility of the conditions using a non-PDC (parallel distributed compensation) controller. Moreover, the work in Guerra and Vermeiren (2004) does not address or consider any form of actuator saturation. Hence, the intention is not to compare the feasibility results for the parameter σ with those of the referenced study, but rather to investigate the effects of saturation on closed-loop stability.*

In conclusion, consider that the membership function $\alpha(k) \in \Gamma_N$ depends on the states, tailoring the model into a T-S fuzzy or *quasi*-LPV approach, as discussed in the previous sections. Following Guerra and Vermeiren (2004), assume $\alpha_1(k) = (\sigma + x_1(k))/(2\sigma)$ and $\alpha_2(k) = 1 - \alpha_1(k)$. Consequently, the estimated region of attraction obtained through (3.32) results in $\xi^\top(k)(P_1\alpha_1(k) + P_2\alpha_2(k))\xi(k) \leq 1$. Incorporating the membership functions into the determination of the region $\mathcal{R}_{\mathcal{E}_a}$ enables capturing additional system information, potentially resulting in an enlarged region. The cut of $\mathcal{R}_{\mathcal{E}_a}$ at $\hat{x} = 0$ for this case with $\sigma = 0.999$ is shown in Figure 3.3: the reader can see the trajectories starting at the boundary of the estimate region of attraction (blue lines), which converge asymptotically to the origin. On the contrary, unstable trajectories with initial conditions outside $\mathcal{R}_{\mathcal{E}_a}$ are depicted (red lines). Moreover, the area of the intersection of the attraction region in this cut is equal to 0.9331, an increase of 23.8% compared to the best result in Table 3.1 when the polytopic model is assumed of LPV-type. Furthermore, despite the stable trajectories leaving the cut of the estimate of the region of attraction, they do not leave the contractive set $\mathcal{R}_{\mathcal{E}_a}$ for $\alpha \in \Omega$. This claim can be verified in Figure 3.4 where the Lyapunov function $V(\xi(k))$ for each stable trajectory is presented, monotonically decreasing and less than 1.

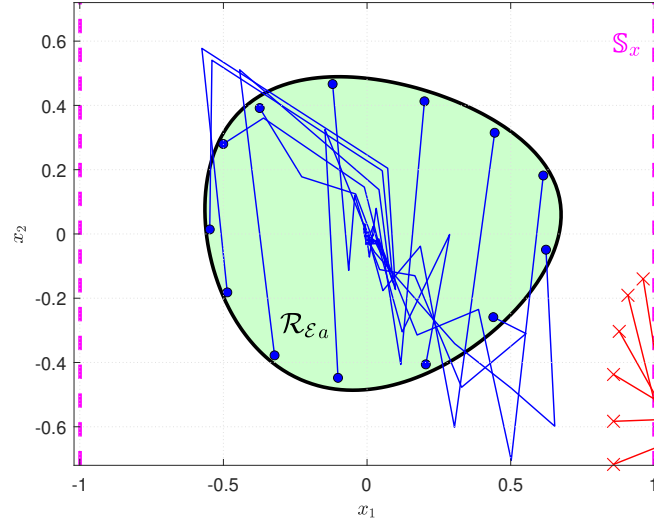


Figure 3.3: Cut of the estimate region of attraction for $\sigma = 0.999$ at $\hat{x} = 0$ and assuming membership functions $\alpha_1(k) = (\sigma + x_1(k))/(2\sigma)$ and $\alpha_2(k) = 1 - \alpha_1(k)$.

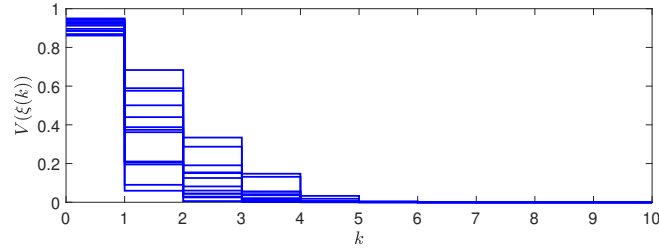


Figure 3.4: Time-behavior of the Lyapunov function for the trajectories with initial conditions belonging to the border of the region of attraction, $\mathcal{R}_{\mathcal{E}a}$, illustrating that all trajectories remain in such set.

3.4.2 Reference Tracking of an LPV Model

The objective of this example is to illustrate the performance of the proposed method for the control of a saturating system under amplitude disturbances and compare the achievements with those reported in the literature. The lateral motion control of an F-4 fighter aircraft, as presented in [Ochi and Kanai \(1995\)](#) and further investigated in [Kucukdemiral et al. \(2022\)](#), is considered. The aircraft actuators are subject to rate and magnitude saturation effects. The

equations (3.7) can describe the F-4 fighter aircraft time-varying model with matrices

$$\begin{aligned}
 A &= \begin{bmatrix} 0.6740 & 1.3676 & -4.1392 & -0.0463 & -0.1089 & 0.7915 \\ 0.0130 & 0.4646 + 0.2\theta(k) & 1.6521 & 0.0173 & -0.1049 & -0.0431 \\ 0.0034 & -0.3894 & 0.4932 & 0.0151 & 0.0592 & 0.0247 \\ 0.4141 & 0.2624 & -1.2543 & 0.9917 & 0.0030 & 0.3213 \\ 0 & 0 & 0 & 0 & 0.0067 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0821 \end{bmatrix}, \\
 B &= \begin{bmatrix} 0.0596 & -1.1801 & 0.2764 & 0.0727 & 1.9865 & 0 \\ 3.2129 & -0.2088 & 0.0509 & 0.6627 & 0 & 1.8358 \end{bmatrix}^\top, \\
 C_z &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad D_z = \begin{bmatrix} 0 \\ 0 \end{bmatrix},
 \end{aligned} \tag{3.43}$$

where the uncertain parameter $|\theta(k)| \leq 1$ varies arbitrarily over time and the matrix C_z selects as outputs the side-slip and roll angles from the aircraft. The aircraft actuators are under symmetric rate and magnitude saturation, with bounds given by $\mu_M = [0.7854 \ 0.5236]^\top$ and $\mu_R = [0.6545 \ 0.5673]^\top$. The dynamic parameters of the actuator are given by $\Lambda = \text{diag}\{1, 1\}$, i.e., an ideal rate limiter, leading to the augmented description given in (3.15)–(3.16).

To align with Kucukdemiral et al. (2022), the reference tracking framework is considered by defining the signal error as $e(k) = r(k) - \hat{y}(k)$, where $r(k)$ represents the reference signal and $\hat{y}(k) = \mathbb{C}(\alpha(k))\xi(k)$ denotes the output of the system with $\mathbb{C}(\alpha(k)) = [C(\alpha(k)) \ 0]$. Thus, the integral control action over the errors is introduced by augmenting the system (3.15)–(3.16) as follows:

$$\begin{aligned}
 \bar{\xi}(k+1) &= \left(\bar{\mathbb{A}}(\alpha(k)) + \bar{\mathbb{B}}_1 \bar{\mathbb{K}}(\alpha(k)) \right) \bar{\xi}(k) + \bar{\mathbb{B}}_1 \Psi_M(k) + \bar{\mathbb{B}}_2 \Psi_R(k) + \bar{\mathbb{B}}_w(\alpha(k))r(k), \\
 \bar{y}(k) &= \bar{\mathbb{C}}(\alpha(k))\bar{\xi}(k), \\
 \bar{z}(k) &= \bar{\mathbb{C}}_z(\alpha(k))\bar{\xi}(k) + \bar{\mathbb{D}}_z(\alpha(k))r(k),
 \end{aligned} \tag{3.44}$$

with

$$\begin{aligned}
 \bar{\mathbb{A}}(\alpha(k)) &= \begin{bmatrix} \mathbb{A}(\alpha(k)) & \mathbf{0} \\ -\mathbb{C}(\alpha(k)) & \mathbf{I} \end{bmatrix}, \quad \bar{\mathbb{B}}_1 = \begin{bmatrix} \mathbb{B}_1 \\ \mathbf{0} \end{bmatrix}, \quad \bar{\mathbb{B}}_2 = \begin{bmatrix} \mathbb{B}_2 \\ \mathbf{0} \end{bmatrix}, \quad \bar{\mathbb{B}}_w(\alpha(k)) = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}, \\
 \bar{\mathbb{C}}(\alpha(k)) &= [\mathbb{C}(\alpha(k)) \ \mathbf{0}], \quad \bar{\mathbb{C}}_z(\alpha(k)) = [-\mathbb{W}_2 \mathbb{C}(\alpha(k)) \ \mathbb{W}_1], \quad \bar{\mathbb{D}}_z(\alpha(k)) = [\mathbb{W}_2].
 \end{aligned} \tag{3.45}$$

Thus, the extended state vector is defined as $\bar{\xi}(k) = [\xi^\top(k) \ e^\top(k)]^\top$ and, by consequence, the controller gain matrices are given by $\bar{\mathbb{K}}(\alpha(k)) = [K(\alpha(k)) \ \hat{K}(\alpha(k)) \ K_I(\alpha(k))]$. Observe that $K_I(\alpha(k))$ refers to the LPV integral control gains. Moreover, the approach admits a full matrix $K_I(\alpha(k)) \in \mathbb{R}^{2 \times 2}$, which is more general than the usual diagonal integral gain. It is important to note that the designer specifically chose the diagonal weighting matrices $\mathbb{W}_1$ and $\mathbb{W}_2$ to achieve the desired system performance. Following Kucukdemiral et al. (2022), the same weights $\mathbb{W}_1 = 0.3 \times \text{diag}\{1/20, 1/20\}$ and $\mathbb{W}_2 = 0.7 \times \text{diag}\{1/20, 1/10\}$ are chosen to ensure a fair comparison.

The objective is to guarantee the stability of the time-varying closed-loop system during the setpoint tracking, ensuring a bound to the ℓ_2/ℓ_∞ gain. In this case, a reference signal is

considered $r(k) = \begin{bmatrix} r_1(k) & r_2(k) \end{bmatrix}^\top$ with

$$r_1(k) = \begin{cases} 0.35, & 6 \leq k < 41, \\ -0.35, & 41 \leq k \leq 80, \\ 0, & \text{otherwise;} \end{cases} \quad r_2(k) = \begin{cases} 0.35, & 6 \leq k \leq 80, \\ 0, & \text{otherwise.} \end{cases} \quad (3.46)$$

Assuming $r(k) = \omega(k)$, from (3.46) and (3.2), it follows that $\beta^{-1} = 0.245$. The optimization procedure $\mathcal{J}_3$ in (3.37) is executed, where the matrices in Theorem 3.1 are replaced by the corresponding ones in the system description (3.44)-(3.45). A coarse grid search over τ_1 and τ_2 was performed to obtain the maximum bound value for the induced ℓ_2/ℓ_∞ gain. By setting $\tau_1 = 0.826$ and $\tau_2 = 29$, the minimum gain of $\sqrt{\gamma} = 0.0403$ was achieved. In contrast, applying the same conditions with (Kucukdemiral et al., 2022, Theorem 6) resulted in a gain that is 98% worse than the proposed approach, i.e., $\sqrt{\gamma} = 0.0799$. This demonstrates the superior performance of the proposed method in this case.

The proposed approach yields the following gain matrices:

$$\begin{bmatrix} \bar{\mathbb{K}}_1 \\ \mathbb{K}_2 \end{bmatrix} = \begin{bmatrix} 0.1354 & 0.5354 & 0.0118 & 0.1748 & -0.1089 & 0.0910 & -0.9724 & 0.3653 & 0.2507 & -0.0263 \\ -0.0853 & -0.1989 & 0.1451 & -0.0915 & 0.0265 & -0.0821 & 0.2362 & -0.3850 & 0.1374 & 0.0096 \\ 0.1227 & 0.7452 & 0.0523 & 0.1575 & -0.1018 & 0.0807 & -1.0515 & 0.3192 & 0.2625 & -0.0238 \\ -0.0838 & -0.2750 & 0.1237 & -0.0893 & 0.0318 & -0.0786 & 0.2150 & -0.3747 & 0.1398 & 0.0093 \end{bmatrix},$$

where the last two columns refer to $K_I(\alpha(k))$. Observe that the control gains of this reference tracking approach retain the algebraic structure of (3.26), which yields a simple parameter-dependent gain implementation scheme. To evaluate the performance of the discrete-time system (3.44) with matrices (3.43), simulations were conducted using the reference tracking signals given by (3.46). The evolution of the side-slip and roll angles of the aircraft, along with their reference signals, is depicted in Figure 3.5, which illustrates their behavior throughout the simulation. It is worth mentioning that the outputs adhere to the desired setpoint, as indicated by the reference signals. In particular, a perturbation in the side-slip angle reference signal ($r_1(k)$) at sample $k = 40$ resulted in oscillations in the roll angle ($y_2(k)$). Nevertheless, both outputs are interconnected due to the inherent characteristics of the system dynamics while still maintaining stability. In the simulations of this example, the parameters related to the uncertainty $\theta(k)$ vary in time according to $\alpha_1(k) = |\sin(0.1k)|$ and $\alpha_2(k) = 1 - \alpha_1(k)$. Figure 3.6 presents the corresponding control signals where the saturation limits are respected.

This example shows how the proposed method can be used in reference tracking, ensuring that the actuator respects the saturating limits. Moreover, the results suggest improved performance of the proposed method compared to a recent approach in the literature.

3.4.3 Real-Time Control of a *quasi*-LPV System

The efficacy of the proposed method is demonstrated through a real-time experiment using a nonlinear tank system operated by conventional industrial devices, which serves as a benchmark in related studies (Lopes et al., 2020b; Oliveira et al., 2020; Quadros et al., 2020; Ferreira et al., 2022). The following sections present the system description, its *quasi*-LPV modeling, and, finally, the controller design acting in a real-time experiment.

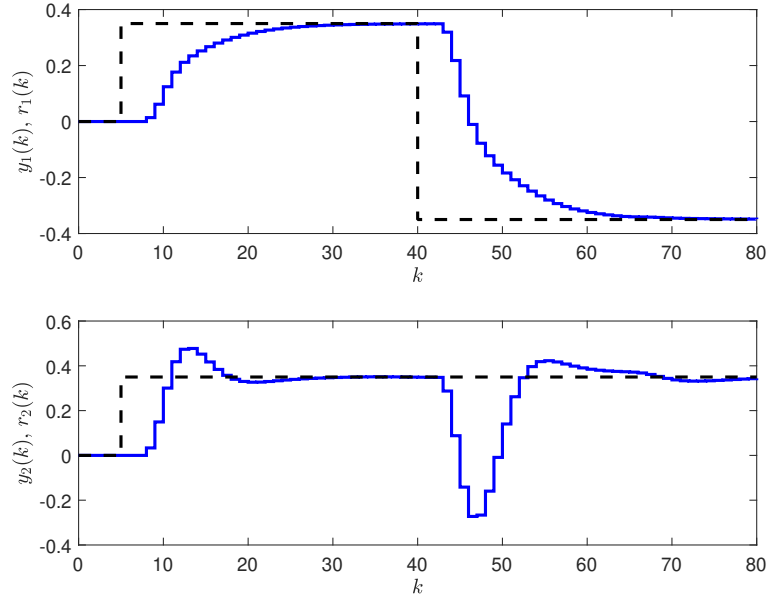


Figure 3.5: Outputs of the system (blue lines) tracking the references signals (black dashed-lines). A change in the side-slip angle causes a deviation in the roll angle due to the system dynamics, but the integral controller acts to correct it.

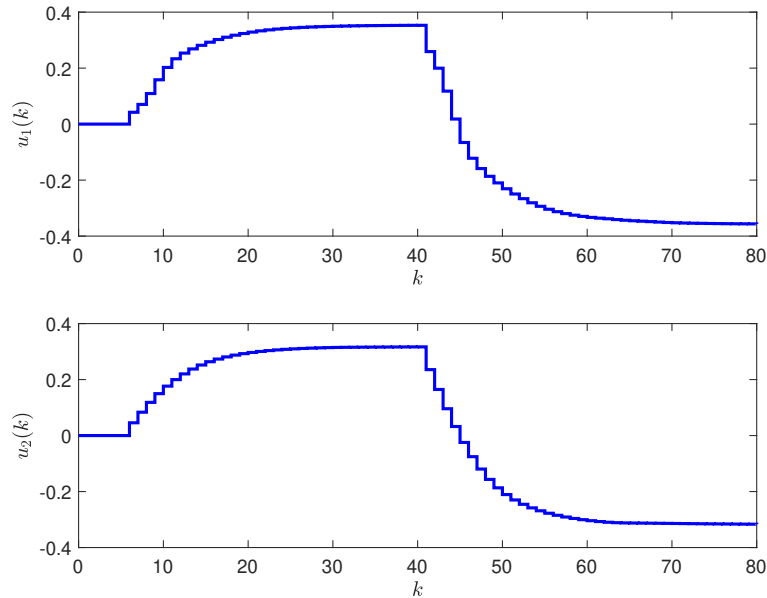


Figure 3.6: Control signals applied to the F-4 fighter aircraft model, where the saturation bounds are respected.

Description of the system

The experimental setup comprises two tanks, designated as $T3$ and $T4$, each with a capacity of 200 liters. Figure 3.7 provides an illustrative schematic diagram depicting the configuration of the system. Tanks $T3$ and $T4$ are interconnected by a manual valve that remains fixed throughout the experiment. Differential pressure sensors, specifically the Honeywell model 26PCBFA6D, are employed to measure the water levels $h_1(t)$ and $h_2(t)$ in the tanks. Notably, inside tank $T3$, a nonlinear solid is inserted, assuming a Gaussian curve revolution shape and made of expanded

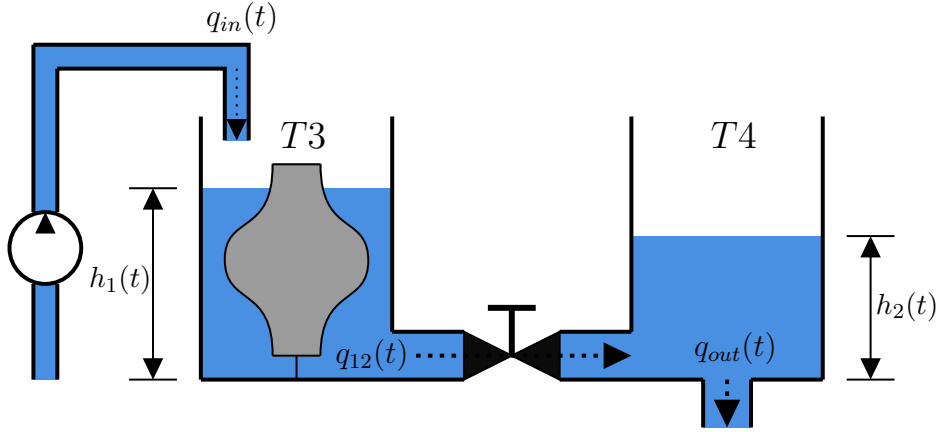


Figure 3.7: Diagram of the coupled tank system used to perform a real-time experiment. The water flows from $T3$ to $T4$, recirculated by a pump motor. Tank $T3$ has a structure inside, yielding nonlinear dynamics.

polystyrene. Consequently, the volume of liquid within tank $T3$ is not directly proportional to its height, introducing significant nonlinear dynamics to the system. Thus, this interconnected system is modeled as a *quasi*-LPV system.

The experimental setup supplies water from two 400-liter reservoirs to tank $T3$ via the inflow $q_{in}(t)$ using a 1-horsepower motor pump manufactured by Dancor, model CAM-W6. Subsequently, the fluid flows from tank $T3$ to tank $T4$ through their connecting pathway ($q_{12}(t)$). Finally, the outflow ($q_{out}(t)$) in tank $T4$ directs the water back to the reservoirs, where the motor pump recirculates it.

The control system integrates an open-source Python interface (Sousa et al., 2018) that interacts with a Siemens Simatic $S7 - 300$ programmable logic controller (PLC). The PLC monitors the sensor signals and transmits control commands to a WEG $CFW09$ frequency inverter that, in turn, regulates the power supplied to the pump following the decisions made by the controller.

Quasi-LPV Modeling

The following sections present the physical modeling of the system, its discretization, and the corresponding *quasi*-LPV model cast in a polytopic description as shown in (3.7). Subsequently, the proposed method is applied to design a controller that accounts for the rate and magnitude saturation limits of the actuator. Finally, a real-time level control experiment is conducted to demonstrate a complete application of the methodology.

The mathematical modeling of the level dynamics is based on mass balance equations, yielding

$$\begin{aligned} \dot{h}_1(t) &= \frac{q_{in}(t)}{A_1(h_1(t))} - \frac{h_1(t) - h_2(t)}{R_{12}(h_1(t), h_2(t))A_1(h_1(t))}, \\ \dot{h}_2(t) &= \frac{h_1(t) - h_2(t)}{R_{12}(h_1(t), h_2(t))A_2} - \frac{q_{out}(t)}{A_2}, \end{aligned} \quad (3.47)$$

where $h_1(t)$ represents the level of tank $T3$ and $h_2(t)$ represents the level of tank $T4$, the flow

functions are experimentally determined as follows:

$$q_{in}(t) = 15.0953\hat{u}(t) \text{ cm}^3/\text{s},$$

where the input flow is controlled by the signal $\hat{u}(t)$, and

$$q_{out}(t) = -0.1758h_2(t)^2 + 20.6553h_2(t) + 222.5698 \text{ cm}^3/\text{s},$$

representing the output flow in tank $T4$. The flow resistance between the tanks is modeled as

$$R_{12}(h_1(t), h_2(t)) = (8.4874 + 1.4297(h_1(t) - h_2(t)) - 0.0324(h_1(t) - h_2(t))^2) \times 10^{-3} \text{ s/cm}^2.$$

The transversal area of tank $T4$ is constant and given by $A_2 = 3019 \text{ cm}^2$, while the corresponding area in tank $T3$ exhibits a nonlinear dependency on $h_1(t)$ as follows:

$$A_1(h_1(t)) = \frac{3r}{5} \left(2.7r - \frac{\cos(2.5\pi((h_1(t) - 8) \times 10^{-2} - \mu))}{\omega\sqrt{2\pi}} \times e^{-\frac{((h_1(t)-8) \times 10^{-2} - \mu)^2}{2\omega^2}} \right) \times 10^4 \text{ cm}^2,$$

with $r = 0.31 \text{ m}$, $\sigma = 0.55$, and $\mu = 0.4$. Following previous studies, a sampling time of $T = 4$ seconds is used to discretize model (3.47) via forward Euler discretization method, resulting in:

$$\begin{bmatrix} h_1(k+1) \\ h_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 - z_1(k)z_3(k) & z_1(k)z_3(k) \\ \frac{z_1(k)}{A_2} & 1 - \frac{z_1(k) + z_2(k)}{A_2} \end{bmatrix} \begin{bmatrix} h_1(k) \\ h_2(k) \end{bmatrix} + \begin{bmatrix} TK_b z_3(k) \\ 0 \end{bmatrix} \hat{u}(k), \quad (3.48)$$

with the functions given by

$$z_1(k) = \frac{T}{R_{12}(h_1(k), h_2(k))}, \quad z_2(k) = \frac{Tq_{out}}{h_2(k)}, \quad \text{and} \quad z_3(k) = \frac{1}{A_1(h_1(k))}.$$

The operational intervals $22.5 \leq h_1 \leq 46.5 \text{ cm}$ and $12.46 \leq h_2 \leq 32.46 \text{ cm}$ are considered, and consequently, the ranges of the functions $z_i(k)$ are obtained as

$$\begin{aligned} 180.164 &\leq z_1(k) \leq 471.141, \\ 83.682 &\leq z_2(k) \leq 139.394, \text{ and} \\ 36.606 \times 10^{-5} &\leq z_3(k) \leq 150.595 \times 10^{-5}. \end{aligned}$$

Employing the sector nonlinearity approach (Tanaka and Wang, 2001), it follows that $z_i(k) = w_i^1(k)\bar{z}_i + w_i^2(k)z_i$, $i = 1, \dots, 3$, with $w_i^1(k) \geq 0$, $w_i^2(k) \geq 0$, $w_i^1(k) + w_i^2(k) = 1$, $w_i^1(k) = (\bar{z}_i - z_i(k))/(\bar{z}_i - z_i)$, and $w_i^2(k) = (z_i(k) - z_i)/(\bar{z}_i - z_i)$. This yields a discrete-time *quasi*-LPV model, whose vertices concerning (3.7) with $N = 8$ are given by:

$$\begin{bmatrix} A_1 & A_2 & A_3 & A_4 \\ A_5 & A_6 & A_7 & A_8 \end{bmatrix} = \begin{bmatrix} 1 - z_1\bar{z}_3 & \frac{z_1\bar{z}_3}{A_2} & 1 - \bar{z}_1\bar{z}_3 & \frac{\bar{z}_1\bar{z}_3}{A_2} & 1 - z_1\bar{z}_3 & \frac{z_1\bar{z}_3}{A_2} & 1 - \bar{z}_1\bar{z}_3 & \frac{\bar{z}_1\bar{z}_3}{A_2} \\ \frac{z_1}{A_2} & 1 - \frac{z_1 + z_2}{A_2} & \frac{\bar{z}_1}{A_2} & 1 - \frac{\bar{z}_1 + \bar{z}_2}{A_2} & \frac{z_1}{A_2} & 1 - \frac{z_1 + z_2}{A_2} & \frac{\bar{z}_1}{A_2} & 1 - \frac{\bar{z}_1 + \bar{z}_2}{A_2} \\ 1 - \bar{z}_1\bar{z}_3 & \frac{\bar{z}_1\bar{z}_3}{A_2} & 1 - z_1\bar{z}_3 & \frac{z_1\bar{z}_3}{A_2} & 1 - \bar{z}_1\bar{z}_3 & \frac{\bar{z}_1\bar{z}_3}{A_2} & 1 - z_1\bar{z}_3 & \frac{z_1\bar{z}_3}{A_2} \\ \frac{z_1}{A_2} & 1 - \frac{z_1 + z_2}{A_2} & \frac{\bar{z}_1}{A_2} & 1 - \frac{\bar{z}_1 + \bar{z}_2}{A_2} & \frac{z_1}{A_2} & 1 - \frac{z_1 + z_2}{A_2} & \frac{\bar{z}_1}{A_2} & 1 - \frac{\bar{z}_1 + \bar{z}_2}{A_2} \end{bmatrix}, \quad (3.49)$$

with $B_1 = B_3 = B_5 = B_7 = [TK_b \bar{z}_3 \ 0]^\top$, $B_2 = B_4 = B_6 = B_8 = [TK_b z_3 \ 0]^\top$, and the time-varying parameter vector given by $\alpha(k) = [w_1^1(k)w_2^1(k)w_3^1(k) \ \dots \ w_1^2(k)w_2^2(k)w_3^2(k)]$. This results in a discrete-time polytopic description of the experimental setup, enabling the application of the proposed methods.

Controller Design and Experiment

An equilibrium point within the operating range must be selected to run the experiment. Thus, with a control signal of $u^* = 38\%$ of the power of the pump, the experimental values $h_1^* = 34.51$ cm and $h_2^* = 22.46$ cm were obtained. Consequently, the state vector is given as $x(k) = [h_1(k) - h_1^* \quad h_2(k) - h_2^*]^\top$, and the control signal as $u(k) = \hat{u}(k) - u^*$. Following the reference tracking framework presented in (3.44)–(3.45), the output is defined as $y(k) = Cx(k)$, with $C = [1 \quad 0]$, indicating a reference tracking of the $h_1(k)$ level. Rate and magnitude saturation are symmetrically bounded by $\mu_R = 1$ and $\mu_M = 15$, respectively, meaning that $u(k)$ can vary of $\pm 15\%$ around the operational point $u^* = 38\%$ and $-1\% \leq u(k+1) - u(k) = \hat{u}(k+1) - \hat{u}(k) \leq 1\%$ or, in a temporal basis, the actuator rate is constrained by $\pm 0.25\%/s$, since the sampling period is $T = 4$ s. Thus, these choices mean that the control signal $u(k)$ can vary from 23% to 53%, i.e., $-15\% \leq \hat{u}(k) \leq 15\%$, and its change between two samples is constrained to $\pm 1\%$. Note that such a variation limit is a constraint inserted in the frequency inverter device that actuates the motor pump, avoiding, for instance, electrical and mechanical efforts on the motor pump. Additionally, an ideal rate limiter with $\Lambda = 1$ is assumed. The validity domain of the model (3.49) is given by $|x_1(k)| \leq 12$ and $|x_2(k)| \leq 10$, meaning that the level $h_1(k)$ may vary from 22.51 cm up to 46.51 cm. Such a choice leads the system to operate in a range where the nonlinearity in tank $T3$ has stronger effects on the level dynamics. Consequently, by applying equation (3.6) in Remark 3.1, the $\mathbb{S}_x$ -region is characterized with

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

and $\rho = [12 \quad 10]$. Note that, with such a change of coordinates, $r(k) = 0$ means $x_1(k) = 0$, or equivalently $h_1(k) = h_1^*$. Therefore, $r(k)$ denotes the desired variation of $h_1(k)$ around h_1^* .

The objective is to determine the maximum allowable amplitude of $r(k) \in \mathcal{W}$, i.e., to find the minimum β in (3.2), corresponding to the variation of the reference signal $r(k)$, such that the state trajectories do not leave the region of attraction of the equilibrium point (h^*, u^*) . Thus, the optimization procedure $\mathcal{J}_2$ given in (3.36) was used with $\eta = 1$ and $\tau_1 = 0.985$, yielding $\tau_2 = 3.818 \times 10^{-4}$ and the maximum bound for $r(k)$ of $\pm\sqrt{\beta^{-1}} = \pm 6.267$ cm around the equilibrium point h_1^* . The resulting controller gains are implemented in the real system using the parameter-dependent gain structure found in (3.26) for a reference signal respecting the maximum bounds. The achievements are reported in Figure 3.8, where the left-hand side illustrates the time response of the controlled level $h_1(k)$ (top) and the corresponding control signal sent to the motor pump (bottom). The system starts close to the equilibrium condition, and the reference signal is chosen to reach the estimated limit, i.e., $h_1^* \pm \sqrt{\beta^{-1}}$ or equivalently $28.24 \leq h_1(k) \leq 40.78$ cm. The measurement noises come mainly from the water oscillations in the tanks. Figure 3.8 (b) (right-hand side) shows the behavior of the parameter-dependent Lyapunov function (top) and the time-varying parameters of the *quasi*-LPV controlled system (bottom). As expected, the parameter-dependent Lyapunov function starts near zero because the system is close to equilibrium. Moreover, it does not violate the unitary value, meaning that

the state trajectory remains inside the estimate of the region of attraction, $\mathcal{R}_{\mathcal{E}_a}$, respecting the level set. The bottom part of Figure 3.8 (b) shows the behavior of the components of parameters $\alpha(k)$ recovered from the *quasi*-LPV system.

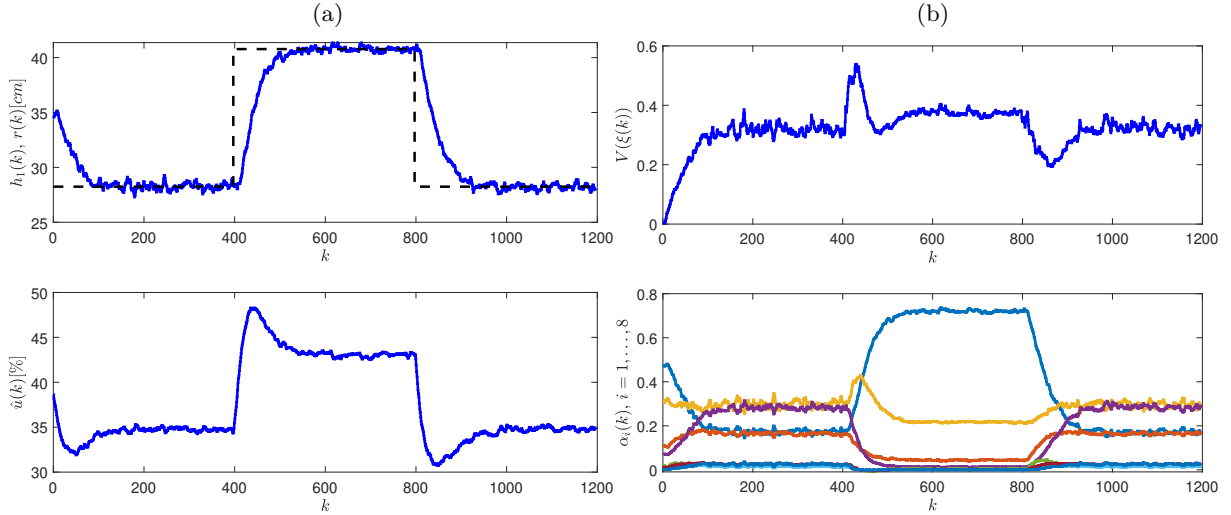


Figure 3.8: Real-time reference track experiment performed with the designed controller: (a) Top plot: time response of level $h_1(k)$ (blue lines) with the respective reference signal (black dashed lines). Bottom plot: time response of the control signal during the experiment. (b) Top plot: behavior of the Lyapunov function, demonstrating that the function never exceeds the unitary value. Bottom plot: time-varying parameters of the *quasi*-LPV model throughout the experiment.

Figure 3.9 shows the experimental data in a phase-plane portrait. The region of attraction $\mathcal{R}_{\mathcal{E}_a}$ has been computed regarding the time-varying functions, as described in (3.32). Note that the phase plane is shown in two dimensions; therefore, the corresponding region of $\mathcal{R}_{\mathcal{E}_a}$ is cut at $\hat{x} = 0$. The black dashed-line region represents the projection of the three-dimensional $\mathcal{R}_{\mathcal{E}_a}$ on the phase plane. Therefore, it can be verified that, as stated in Theorem 3.1, the trajectories of the closed-loop system (blue lines) remain within the set $\mathcal{R}_{\mathcal{E}_a}$, for all $k \geq 0$ once the exogenous signal, $r(k)$, remains in the set $\mathcal{W}$. Since this is a real-time experiment exploring the limits of $r(k)$, the measured signals are subject to noise, which explains some points lying slightly outside the projection region.

In summary, the optimization procedure $\mathcal{J}_2$ was employed to determine the maximum variation of the reference signal ($r(k)$, the exogenous signal) such that the closed-loop trajectories remain within the guaranteed region of attraction for the nonlinear coupled tanks modeled as a polytopic *quasi*-LPV system. The experimental data indicate that the proposed method is effective and suitable for industrial applications.

3.5 Final Considerations

A condition is provided to address discrete-time varying systems under magnitude and rate saturating actuators. The generality of the polytopic representation proves highly advantageous,

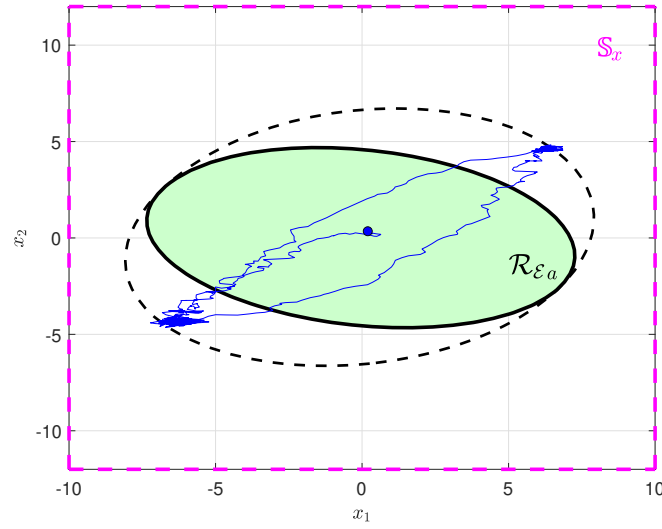


Figure 3.9: Phase plane portrait with experimental data (blue line), the cut of the estimate of the region of attraction $\mathcal{R}_{\mathcal{E}_a}$ at $\hat{x}(k) = 0$ (solid black line), and the projection of $\mathcal{R}_{\mathcal{E}_a}$ (dashed black line), all of them included in the region of validity of the model, $\mathbb{S}_x$ (dashed magenta line).

as it accommodates a broad range of models, including LPV, *quasi*-LPV, and Takagi–Sugeno (T-S) fuzzy models. Both rate and magnitude saturations are considered, leading to a characterization of the set of initial conditions for which system trajectories converge to the equilibrium point (in the absence of exogenous disturbances). The controller employs a state feedback gain parameterized by system variables, ensuring local stabilization of the saturated system. Additionally, exogenous disturbance signals with bounded amplitude are incorporated, with an estimation of the ℓ_2/ℓ_∞ gain between the disturbance input and the regulated output. Consequently, this approach is capable of addressing various control problems, including reference tracking. Optimization procedures are proposed to design local stabilizing control gains while maximizing the estimated region of attraction, allowable disturbance levels, or minimizing the ℓ_2/ℓ_∞ gain. A set of examples is provided to illustrate the effectiveness of the approach and to benchmark against recent literature. Moreover, the methodology is demonstrated in the context of polytopic LPV and T-S fuzzy modeling. Experimental validation is performed through a real-time study involving a nonlinear process modeled as a *quasi*-LPV system, where the actuator is a motor pump subject to magnitude and slew rate constraints.

New Negative Definiteness Conditions for Quadratic Functions in LPV Systems

Diego no conocía la mar. El padre, Santiago Kovadloff, lo llevó a descubrirla. Viajaron al sur. Ella, la mar, estaba más allá de los altos médanos, esperando. Cuando el niño y su padre alcanzaron por fin aquellas cumbres de arena, después de mucho caminar, la mar estalló ante sus ojos. Y fue tanta la inmensidad de la mar, y tanto su fulgor, que el niño quedó mudo de hermosura. Y cuando por fin consiguió hablar, temblando, tartamudeando, pidió a su padre: — “¡Ayúdame a mirar!”

El Libro de los Abrazos (1989)
EDUARDO GALEANO

This chapter addresses negative definiteness conditions of quadratic functions, common in control problems with time-varying delay systems. Although geometric conditions can be relaxed by partitioning techniques, these relaxations do not necessarily guarantee improved or monotonic performance, and their degree of optimality therefore remains uncertain. Alternative conditions based on the generalized S-procedure are known to be necessary and sufficient when coefficients are not dependent on uncertain parameters; otherwise, they are sufficient only. New approaches are proposed to mitigate these issues, which have not been explored in previous studies, as demonstrated in the stability analysis of LPV sampled-data control systems. First, quadratic polynomial inequalities are rewritten as homogeneous ones, and less conservative conditions are derived using Young’s inequality or Pólya’s theorem, which provide recursive, monotonic, and convergent relaxations. Thereafter, based on Bézier curve equivalence and the de Casteljau algorithm, less conservative recursive monotonic convergent conditions are obtained. Two numerical examples illustrate the effectiveness and improvements over previous related works.

4.1 Recent Results and Problem Formulation

Recall the quadratic polynomial inequality presented in Section 2.3.1. Consider quadratic polynomial matrix inequalities given by:

$$\mathcal{P}(\tau) = \tau^2 \Pi_2 + \tau \Pi_1 + \Pi_0 < \mathbf{0}, \forall \tau \in [\underline{\tau}, \bar{\tau}], \quad (4.1)$$

where $\Pi_i \in \mathbb{S}^p$ ($i = 0, 1, 2$) and τ is a positive real bounded scalar quantity. Such constraints play an important role in control theory, especially for stability analysis of dynamic systems with time-varying delays (see, e.g., Zhang et al. (2022b); Chen et al. (2022b)). However, since these matrix inequalities may not be convex in τ , ongoing research focuses on providing conditions based on LMIs to satisfy (4.1) with minimal conservatism. Widely used approaches involve geometrical considerations to establish sufficient conditions, summarized in the following lemmas, which include the pioneer and most recent related results from the literature; see, e.g., Kim (2011); Yang and Zhang (2014); Kim (2016); Yang et al. (2019); Chen et al. (2019); Zhang et al. (2020a); Liu et al. (2023) and the surveys in Zhang et al. (2022b); Chen et al. (2022b); He et al. (2023) for a complete overview.

Lemma 4.1. *Let $\Delta\tau = \bar{\tau} - \underline{\tau}$. The quadratic inequality (4.1) holds if one of the following sets of conditions is satisfied.*

- i)* (Kim, 2011; Yang and Zhang, 2014) $\mathcal{P}(\underline{\tau}) < \mathbf{0}$, $\mathcal{P}(\bar{\tau}) < \mathbf{0}$ and $\Pi_2 \geq \mathbf{0}$;
- ii)* (Kim, 2016; Yang et al., 2019) $\mathcal{P}(\underline{\tau}) < \mathbf{0}$, $\mathcal{P}(\bar{\tau}) < \mathbf{0}$ and $\mathcal{P}(\bar{\tau}) - \Delta\tau^2\Pi_2 < \mathbf{0}$;
- iii)* (Zhang et al., 2020a) $\exists \mu \in [0, 1]$ such that $\mathcal{P}(\underline{\tau}) < \mathbf{0}$, $\mathcal{P}(\bar{\tau}) < \mathbf{0}$, $\mathcal{P}(\underline{\tau}) - \mu^2\Delta\tau^2\Pi_2 < \mathbf{0}$ and $\mathcal{P}(\bar{\tau}) - (1 - \mu)^2\Delta\tau^2\Pi_2 < \mathbf{0}$;
- iv)* (Liu et al., 2023) $\exists \mu \in [0, 1]$ such that $\mathcal{P}(\underline{\tau}) < \mathbf{0}$, $\mathcal{P}(\bar{\tau}) < \mathbf{0}$, $\mathcal{P}(\underline{\tau}) + \mathcal{P}(\underline{\tau} + \mu\Delta\tau) - \mu^2\Delta\tau^2\Pi_2 < \mathbf{0}$ and $\mathcal{P}(\bar{\tau}) + \mathcal{P}(\bar{\tau} + \mu\Delta\tau) - (1 - \mu)^2\Delta\tau^2\Pi_2 < \mathbf{0}$.

In Lemma 4.1, conditions *i)* require convexity (in τ) of the scalar polynomial $\xi^\top \mathcal{P}(\tau) \xi$, $\forall \xi \in \mathbb{R}^p \setminus \{0\}$, often overly conservative. Therefore, less conservative conditions *ii)* were introduced to accommodate convex or concave polynomials. To further relax these, conditions *iii)* and *iv)* incorporate a parameter μ that must be tuned within $[0, 1]$ or treated as a decision variable, leading to bilinear matrix inequality (BMI) constraints, making the application for computationally complex control problems harder. It is important to note that these geometrical conditions are only sufficient. Hence, improvements have been recently proposed considering partitioning approaches (Chen et al., 2019; He et al., 2023; Liu et al., 2023), summarized in the following lemma, which consists in splitting the interval of the definition of τ into $N_p \in \mathbb{N}^*$ equally spaced partitions, i.e., $[\underline{\tau}, \bar{\tau}] = \cup_{i=1}^{N_p} [\underline{\tau}_i, \bar{\tau}_i]$ with:

$$\underline{\tau}_1 = \underline{\tau}, \bar{\tau}_i = \underline{\tau}_{i+1} = \underline{\tau} + i \frac{\Delta\tau}{N_p}. \quad (4.2)$$

In the sequel, $\mathbb{I}_{N_p}$ denotes the set of positive integers from 0 to N_p and $\mathbb{I}_{N_p}^* = \mathbb{I}_{N_p} \setminus \{0\}$.

Lemma 4.2. *For a given $N_p \in \mathbb{N}^*$, the quadratic inequality (4.1) holds if one of the following sets of conditions is satisfied, $\forall i \in \mathbb{I}_{N_p}^*$.*

- i)* (Chen et al., 2019) $\mathcal{P}(\underline{\tau}) < \mathbf{0}$, $\mathcal{P}(\bar{\tau}_i) < \mathbf{0}$, and $\mathcal{P}(\bar{\tau}_{i-1}) - \frac{\Delta\tau^2}{N_p^2}\Pi_2 < \mathbf{0}$;
- ii)* (He et al., 2023) $\mathcal{P}(\underline{\tau}) < \mathbf{0}$, $\mathcal{P}(\bar{\tau}) < \mathbf{0}$, and $\mathcal{P}\left(\bar{\tau}_i - \frac{\Delta\tau}{2N_p}\right) - \frac{\Delta\tau^2}{4N_p^2}\Pi_2 < \mathbf{0}$;

iii) (*Liu et al., 2023*) $\mathcal{P}(\underline{\tau}) < \mathbf{0}$, $\mathcal{P}(\bar{\tau}) < \mathbf{0}$, and $\mathcal{P}(\bar{\tau}_{i-1}) + \mathcal{P}(\bar{\tau}_i) - \frac{\Delta\tau^2}{N_p^2}\Pi_2 < \mathbf{0}$.

These partitioning approaches offer recursive relaxations and might approximate necessary and sufficient conditions for large N_p , when Π_0 , Π_1 , and Π_2 are constant matrices. However, they suffer from the following fact (illustrated by a numerical example given in Section 4.4).

Fact 4.1. *With the above considered partitioning approaches, there is no guarantee that, if a solution exists for a given N_p , then there is a solution for $N_p + 1$.*

Therefore, these partitioning-based conditions are said to be non-monotonic convergent (non-MC) as N_p increases, which hinders the search for a large enough N_p to closely approximate the optimal solution.

In addition to extensive research efforts to relax geometrical conditions, recent studies established milestone results through the application of the generalized S-procedure (*de Oliveira and Souza, 2020; Chen et al., 2022a; Park and Park, 2020; Kim, 2021; Rouamel et al., 2023*), summarized in the next lemma.

Lemma 4.3. (*Park and Park, 2020; Kim, 2021; Rouamel et al., 2023*)

The quadratic inequality (4.1) holds if there exists $\mathbf{0} \leq M + M^\top \in \mathbb{R}^{p \times p}$ such that:

$$\begin{bmatrix} \mathcal{P}(\tau) & \frac{1}{2}\Pi_1 + \tau\Pi_2 + \Delta\tau M \\ \star & \Pi_2 - M - M^\top \end{bmatrix} < \mathbf{0}.$$

Lemma 4.3 has the advantage of being necessary and sufficient when Π_0 , Π_1 , and Π_2 are parameter-independent matrices, without the need for partitioning approaches. However, it requires the introduction of additional decision variables. As claimed by many authors of recent studies (see, e.g., *Zeng et al. (2020); Chen and Li (2021); Zhang et al. (2022b); Chen et al. (2022b); He et al. (2023)*), this can be highly demanding and impose significant computational burden, particularly for large matrix inequalities with numerous decision variables, as commonly involved in the time-delay systems framework (e.g., when applying Jensen, Wirtinger or Bessel-Legendre inequalities (*Seuret and Gouaisbaut, 2015; Liu et al., 2017; Kim, 2021; Lopes et al., 2021; Rouamel et al., 2023*)).

Fact 4.2. *If the inequality (4.1) involves uncertain parameter-dependent matrices $\Pi_0(\alpha)$, $\Pi_1(\alpha)$, and $\Pi_2(\alpha)$, where $\alpha \in \mathbb{R}^p$ with bounded entries $\alpha_i \in [\underline{\alpha}_i, \bar{\alpha}_i]$, $\forall i \in \mathbb{I}_p^*$, then the conditions in Lemmas 4.2 and 4.3 are no longer necessary, but only sufficient.*

Furthermore, to reduce the number of decision variables (i.e., to lower computational complexity) compared to Lemma 4.3 (*Park and Park, 2020; Kim, 2021; Rouamel et al., 2023*), matrix injection approaches were used in Lemma 4 of *Xie et al. (2023)*. Nonetheless, these conditions also suffer from Fact 4.2.

According to Facts 4.1 and 4.2, it appears that there is still room for improvement. Therefore, the problem statement of this chapter can be summarized as follows.

Problem 4.1. *Provide less conservative negative definiteness conditions for (4.1), which are (when possible) monotonic convergent (MC) as N_p (or any other recursive parameter) increases to an optimal solution (i.e., ensure that if a solution exists for N_p , then it also exists for $N_p + 1$).*

This chapter addresses this problem by introducing two previously unexplored novel approaches. The first considers rewriting $\mathcal{P}(\tau)$ in (4.1) as a homogeneous polynomial matrix inequality (see e.g., Oliveira and Peres (2005)), then applying recursive relaxations from the application of Young's inequality or Pólya's Theorem (Hardy et al., 1952). The second alternative consists of rewriting $\mathcal{P}(\tau)$ in (4.1) as a Bézier curve, then applying recursive relaxations from the application of de Casteljau's algorithm (Farin, 2002). To the best of the authors' knowledge, investigating the negative definiteness of quadratic functions (4.1) from homogeneous polynomials or Bézier curve approaches has never been done in the literature. Except in a work dealing with Takagi-Sugeno model-based control (Bainier et al., 2024), the use of this approach in providing less conservative LMIs has not been explored so far.

4.2 Main Results

This section presents new non-MC and MC conditions for (4.1) from the perspective of the homogeneous polynomial and Bézier curve approaches, and discusses their computational complexity with respect to previous results.

4.2.1 Homogeneous Polynomial Approach

For any given $N_p \in \mathbb{N}^*$, consider equally spaced partitions of $[\underline{\tau}, \bar{\tau}]$ defined in (4.2). $\forall \tau \in [\underline{\tau}_i, \bar{\tau}_i]$, let $\theta_{1i} = \frac{(\tau - \underline{\tau}_i)N_p}{\Delta\tau} \in [0, 1]$ and $\theta_{2i} = \frac{(\bar{\tau}_i - \tau)N_p}{\Delta\tau} \in [0, 1]$, where $\theta_{1i} + \theta_{2i} = 1$. Hence, $\tau = \theta_{1i}\bar{\tau}_i + \theta_{2i}\underline{\tau}_i$ and (4.1) can be rewritten as:

$$(\theta_{1i}\bar{\tau}_i + \theta_{2i}\underline{\tau}_i)^2\Pi_2 + (\theta_{1i}\bar{\tau}_i + \theta_{2i}\underline{\tau}_i)\Pi_1 + \Pi_0 < \mathbf{0}, \forall i \in \mathbb{I}_{N_p}^*. \quad (4.3)$$

By multiplying the second and the third terms in (4.3) respectively by $\theta_{1i} + \theta_{2i}$ and $(\theta_{1i} + \theta_{2i})^2$, it is equivalent to the second-order homogeneous polynomial inequality:

$$\theta_{1i}^2\mathcal{P}(\bar{\tau}_i) + \theta_{1i}\theta_{2i}\mathcal{F}(\underline{\tau}_i, \bar{\tau}_i) + \theta_{2i}^2\mathcal{P}(\underline{\tau}_i) < \mathbf{0}, \forall i \in \mathbb{I}_{N_p}^*, \quad (4.4)$$

with $\mathcal{F}(\underline{\tau}_i, \bar{\tau}_i) = 2\underline{\tau}_i\bar{\tau}_i\Pi_2 + (\underline{\tau}_i + \bar{\tau}_i)\Pi_1 + 2\Pi_0$.

Considering the well-known Young's inequality, the next theorem provides less conservative conditions for (4.4), and so (4.1).

Theorem 4.1. *For a given $N_p \in \mathbb{N}^*$, the quadratic inequality (4.1) holds if $\mathcal{P}(\bar{\tau}_i) < \mathbf{0}, \forall i \in \mathbb{I}_{N_p}$, $2\mathcal{P}(\bar{\tau}_i) + \mathcal{F}(\underline{\tau}_i, \bar{\tau}_i) < \mathbf{0}$ and $2\mathcal{P}(\underline{\tau}_i) + \mathcal{F}(\underline{\tau}_i, \bar{\tau}_i) < \mathbf{0}, \forall i \in \mathbb{I}_{N_p}^*$.*

Proof: When $\mathcal{F}(\underline{\tau}_i, \bar{\tau}_i) < \mathbf{0}$, the inequalities $\mathcal{P}(\bar{\tau}_i) < \mathbf{0}, \forall i \in \mathbb{I}_{N_p}$, provide the negativeness of (4.4) (and so (4.1)). In the other case, when $\mathcal{F}(\underline{\tau}_i, \bar{\tau}_i) \geq \mathbf{0}$, Young's inequality yield $0 \leq \theta_{1i}\theta_{2i} \leq \frac{1}{2}(\theta_{1i}^2 + \theta_{2i}^2)$. Hence, (4.4) holds if $\theta_{1i}^2(\mathcal{P}(\bar{\tau}_i) + \frac{1}{2}\mathcal{F}(\underline{\tau}_i, \bar{\tau}_i)) + \theta_{2i}^2(\mathcal{P}(\underline{\tau}_i) + \frac{1}{2}\mathcal{F}(\underline{\tau}_i, \bar{\tau}_i)) < \mathbf{0}$, i.e., if the conditions in Theorem 4.1 hold. ■

Theorem 4.1 also suffers from Fact 4.1 and thus provides non-MC conditions for (4.1) as N_p increases (see the numerical example given in Section 4.1). Furthermore, applying Young's inequality makes it only sufficient. The following Theorem provides an alternative based on the application of Pólya's Theorem (Hardy et al., 1952).

Theorem 4.2. *For a given $N_p \in \mathbb{N}^*$ and for a sufficiently large $d \in \mathbb{N}$, the quadratic inequality (4.1) holds if, $\forall i \in \mathbb{I}_{N_p}^*$, the coefficients of the monomials of*

$$(\theta_{1i}^2 \mathcal{P}(\bar{\tau}_i) + \theta_{1i} \theta_{2i} \mathcal{F}(\bar{\tau}_i, \bar{\tau}_i) + \theta_{2i}^2 \mathcal{P}(\bar{\tau}_i))(\theta_{1i} + \theta_{2i})^d \quad (4.5)$$

are all strictly negative.

Proof: Straightforward by applying Pólya's Theorem (Hardy et al., 1952, Theorem 56) on (4.4), since $(\theta_{1i} + \theta_{2i})^d = 1, \forall d \in \mathbb{N}$. ■

An important milestone of Pólya's Theorem is, conversely, if all the monomial coefficients of (4.5) are negative for a large enough d , then it is necessary and sufficient to prove that (4.4) is negative (Oliveira and Peres, 2005). Hence, the conditions in Theorem 4.2 are MC and tend to an optimal solution (when it holds) as d increases (with fixed N_p).

Remark 4.1. *To implement the conditions of Theorem 4.2 into typical convex optimization tools, one can expand (4.5) on the canonical basis of a homogeneous polynomial of order $d + 2$, like in Oliveira and Peres (2005). This process is revisited for clarity in Appendix A, where it is shown that, if the inequalities (A.4), (A.5), and (A.6) hold, then (4.5) is negative-definite (so (4.1) is verified).*

4.2.2 Bézier Curves Approach

Let $\theta = \frac{\tau - \bar{\tau}}{\Delta\tau} \in [0, 1]$. Then, the inequality (4.1) can be rewritten as:

$$P(\theta) = B_2^2(\theta)P_2^0 + B_1^2(\theta)P_1^0 + B_0^2(\theta)P_0^0 < \mathbf{0}, \quad (4.6)$$

where $B_2^2(\theta) = \theta^2$, $B_1^2(\theta) = 2\theta(1-\theta)$, and $B_0^2(\theta) = (1-\theta)^2$ are second-order Bernstein polynomials in θ . $P_2^0 = \mathcal{P}(\bar{\tau})$, $P_1^0 = \frac{1}{2}\mathcal{F}(\bar{\tau}, \bar{\tau})$, and $P_0^0 = \mathcal{P}(\bar{\tau})$ provide the control points of a quadratic Bézier curve (see Farin (2002) for more details) by congruence with any $\xi \in \mathbb{R}^p \setminus \{0\}$. Figure 4.1 shows four examples of four Bézier curves and their respective control points.

Remark 4.2. *The quadratic Bézier representation (4.6) of (4.1) is equivalent to its homogeneous polynomial representation (4.4) with $N_p = 1$. The only difference remains in the weight of the monomials, i.e., $B_1^2(\theta) = 2\theta(1-\theta)$ in (4.6) and $\theta_1\theta_2$ in (4.4). This slight difference provides an important feature for Bernstein polynomials, which belong to the unit simplex while the monomials of homogeneous polynomials (4.4) do not, i.e., $\forall \theta_1 \in [0, 1]$ and $\theta_2 = 1 - \theta_1$, $\theta_1^2 + \theta_1\theta_2 + \theta_2^2 \leq 1$ but not always equal to 1.*

Bézier curves always remain within the convex hull formed by their control points, leading to straightforward conditions for the negative definiteness of (4.1) if all control points in (4.6) are negative (i.e., $P_2^0 < \mathbf{0}$, $P_1^0 < \mathbf{0}$, and $P_0^0 < \mathbf{0}$), which are equivalent to the conditions of Theorem 4.2 with $N_p = 1$ and $d = 0$. Furthermore, a single Bezier curve can be split into two connected curves at a common control point using de Casteljau's algorithm and recursively divided into $2^{\mathcal{N}}$ curves, where $\mathcal{N}$ denotes the number of iterations of de Casteljau's algorithm, with tighter control points approaching the original curve (Farin, 2002). Leveraging these properties, the following theorem provides new conditions to guarantee the negativeness of (4.1).

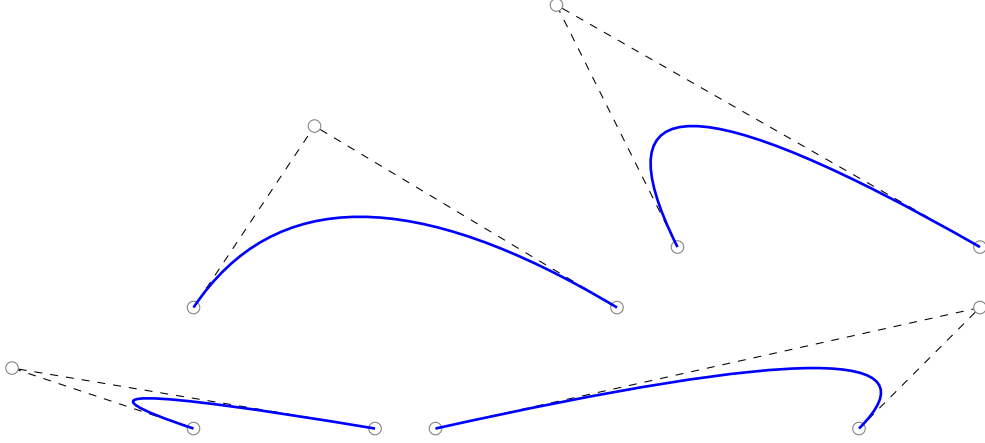


Figure 4.1: Examples of quadratic Bézier curves in blue and their control points represented by the gray dots. See animated following this [link](#).

Theorem 4.3. For a given $\mathcal{N} \in \mathbb{N}^*$, the quadratic inequality (4.1) holds if, $\forall k \in \mathbb{I}_{2(\mathcal{N}-1)-1}$:

$$(e_q^\top \otimes I) \mathbb{P}_k^\mathcal{N} < 0, \forall q \in \mathbb{I}_5^*, \quad (4.7)$$

where $\otimes$ denotes the Kronecker product, $\mathbb{P}_k^\mathcal{N} = (\tilde{D} \prod_{j=0}^{n_b} D_{k_j} \otimes I) \mathbb{P}_0$, where $(k_{n_b} \dots k_j \dots k_0)_2$ denotes the base-2 decomposition of the integer k with $k_j \in \mathbb{I}_1$, e_q ($\forall q \in \mathbb{I}_5^*$) denotes the standard Euclidean basis vectors of $\mathbb{R}^5$, $\mathbb{P}_0^\top = [\mathbb{P}_0^{0\top} \quad \mathbb{P}_1^{0\top} \quad \mathbb{P}_2^{0\top}]$, and:

$$D_0 = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}, D_1 = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}, \tilde{D} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}^\top.$$

Proof: Applying de Casteljau's algorithm to the quadratic Bézier representation (4.6), the Bézier segments are divided into half points without loss of generality for numerical convenience (Farin, 2002). For the first iteration ($\mathcal{N} = 1$), it yields the definition of new control points $\mathbb{P}_0^1$, $\mathbb{P}_1^1$ and $\mathbb{P}_2^1$ for the subdivided Bézier curve on the left, and $\mathbb{P}_2^1$, $\mathbb{P}_3^1$ and $\mathbb{P}_4^1$ for the right one, such that $[\mathbb{P}_0^{1\top} \quad \mathbb{P}_1^{1\top} \quad \mathbb{P}_2^{1\top}] = ((D_0 \otimes I) \mathbb{P}_0)^\top$ and $[\mathbb{P}_2^{1\top} \quad \mathbb{P}_3^{1\top} \quad \mathbb{P}_4^{1\top}] = ((D_1 \otimes I) \mathbb{P}_0)^\top$, where D_0 and D_1 are de Casteljau's matrices for quadratic Bézier curves (Farin, 2002). Since the subdivided left and right Bézier representations share the same control point $\mathbb{P}_2^1$, they can be rearranged as $\mathbb{P}_0^{1\top} = [\mathbb{P}_0^{1\top} \quad \mathbb{P}_1^{1\top} \quad \mathbb{P}_2^{1\top} \quad \mathbb{P}_3^{1\top} \quad \mathbb{P}_4^{1\top}] = ((\tilde{D} \otimes I) \mathbb{P}_0)^\top$. For the second iteration ($\mathcal{N} = 2$), the same procedure is applied to each left and right subdivision, yielding $\mathbb{P}_0^2 = (\tilde{D} D_0 \otimes I) \mathbb{P}_0$ and $\mathbb{P}_1^2 = (\tilde{D} D_1 \otimes I) \mathbb{P}_0$. For the third iteration ($\mathcal{N} = 3$), the resulting control points are $\mathbb{P}_0^3 = (\tilde{D} D_0 D_0 \otimes I) \mathbb{P}_0$, $\mathbb{P}_1^3 = (\tilde{D} D_0 D_1 \otimes I) \mathbb{P}_0$, $\mathbb{P}_2^3 = (\tilde{D} D_1 D_0 \otimes I) \mathbb{P}_0$ and $\mathbb{P}_3^3 = (\tilde{D} D_1 D_1 \otimes I) \mathbb{P}_0$. Finally, by recurrence to the $\mathcal{N}$ -th iteration, the control points are stored element-wise in $\mathbb{P}_k^\mathcal{N}$ (for all $k \in \mathbb{I}_{2(\mathcal{N}-1)-1}$), with negativeness guaranteed if the inequalities in (4.7) are satisfied. ■

Figure 4.2 illustrates one iteration of de Casteljau's algorithm ($\mathcal{N} = 1$), which forms the basis of Theorem 4.3. In the left plot, the original quadratic Bézier curve (blue) is defined by the control points $\mathbb{P}_0^0$, $\mathbb{P}_1^0$, and $\mathbb{P}_2^0$. The first step of the algorithm consists of performing linear interpolations between consecutive control points. In this example, with $\lambda = 0.5$, the intermediate points are given by $Q_0 = (1 - \lambda)\mathbb{P}_0^0 + \lambda\mathbb{P}_1^0$ and $Q_1 = (1 - \lambda)\mathbb{P}_1^0 + \lambda\mathbb{P}_2^0$. A second

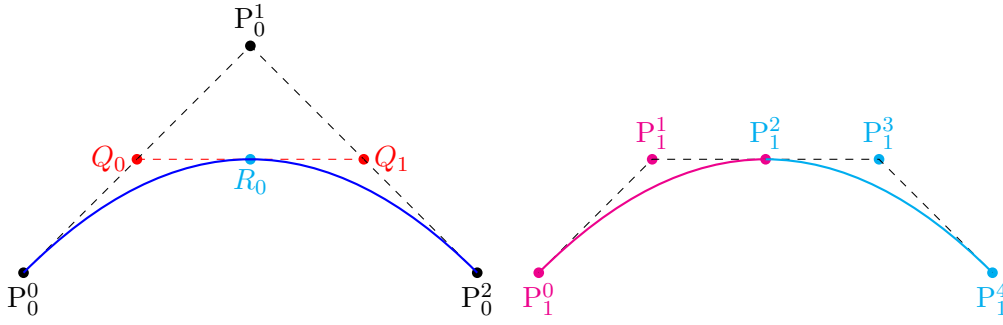


Figure 4.2: Demonstration of de Casteljau's algorithm for a Bézier curve. From an original Bézier curve, two subcurves are obtained with their respective control points.

interpolation between these points yields $R_0 = (1 - \lambda)Q_0 + \lambda Q_1$, which corresponds to the point on the original Bézier curve associated with the parameter value λ .

As shown in the right plot, this construction naturally provides two subdivided Bézier curves. The left subcurve (magenta) is defined by the control points P_1^0 , P_1^1 , and P_1^2 , whereas the right subcurve (cyan) is defined by P_1^2 , P_1^3 , and P_1^4 . This subdivision procedure can be applied recursively for $\mathcal{N}$ iterations. Consequently, verifying the negativity conditions at the refined control points yields a less conservative result than using only the original control polygon.

Remark 4.3. In Theorem 4.3, $\mathcal{N}$ represents the number of iterations in de Casteljau's algorithm, resulting in $2^{\mathcal{N}}$ equally spaced partitions of (4.6) across the parameter range $\theta \in [0, 1]$. This contrasts with the N_p partitioning approach over the parameter range $\tau \in [\underline{\tau}, \bar{\tau}]$ considered in Lemma 4.2 and Theorems 4.1, 4.2. Moreover, since the obtained de Casteljau's subdivisions are equivalent, together, to the original Bézier representation (4.6), the conditions of Theorem 4.3 are MC as $\mathcal{N}$ increases.

Remark 4.4. Extensions of Theorems 4.2 and 4.3 to deal with higher-order polynomial functions than (4.1) are straightforward by considering their higher-order homogeneous polynomial or Bézier curve equivalences.

4.2.3 Computational Complexity

The number of rows of the resulting matrix inequalities relative to p , N_p , d , and $\mathcal{N}$ are listed in Table 4.1 for the conditions outlined in Lemmas 4.1, 4.2, 4.3, Theorems 4.1, 4.2, and 4.3. Lemma 4.1 presents simple sufficient (S) conditions. In contrast, complexity increases with other results required for non-MC or MC conditions. Lemma 4.3 conditions have a smaller number of rows than non-MC or MC conditions, but introduce p^2 additional decision variables (a_{dv}), which could increase the computational burden, particularly for control problems with numerous decision variables. Furthermore, due to Fact 4.2, Lemma 4.3 can be more conservative than non-MC or MC approaches for some control problems, as illustrated in the sequel with the stability analysis of LPV sampled-data control systems. Theorems 4.2 and 4.3 may also be seen as computationally extensive. However, drawing such a conclusion would be a shortcut, since the effective values of N_p , d , or $\mathcal{N}$ must also be considered to achieve an optimal solution with

the benefit of the MC characteristics, as demonstrated via numerical examples in the following section.

Table 4.1: Number of matrix inequality rows, number of additional decision variables (a_{dv}), and types of negative definiteness conditions in Lemmas 4.1-4.3 and Theorems 4.1-4.3.

Methods:	Rows	a_{dv}	Type
Lemma 4.1, <i>i</i>) (Kim, 2011; Yang and Zhang, 2014)	$3p$	-	S
Lemma 4.1, <i>ii</i>) (Zhang et al., 2020a), <i>iii</i>) (Liu et al., 2023)	$4p$	1	S
Lemma 4.2, <i>i</i>) (Chen et al., 2019; He et al., 2023)	$(2N_p + 1)p$	-	non-MC
Lemma 4.2, <i>ii</i>) (He et al., 2023), <i>iii</i>) (Liu et al., 2023)	$(N_p + 2)p$	-	non-MC
Lemma 4.3 (Park and Park, 2020; Kim, 2021)	$3p$	p^2	S
Theorem 4.1	$(3N_p + 1)p$	-	non-MC
Theorem 4.2	$((2 + d)N_p + 1)p$	-	MC
Theorem 4.3	$5(2^{N-1})p$	-	MC

4.3 Application to LPV Sampled-Data Control

Consider a continuous-time LPV system given by:

$$\dot{x}(t) = A(\alpha(t))x(t) + B(\alpha(t))u(t), \quad (4.8)$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^{n_u}$ are the state and control vectors. Matrices $A(\alpha(t)) \in \mathbb{R}^{n \times n}$ and $B(\alpha(t)) \in \mathbb{R}^{n \times n_u}$ are polytopic with N vertices described by

$$\begin{bmatrix} A(\alpha(t)) & B(\alpha(t)) \end{bmatrix} = \sum_{i=1}^N \alpha_i(t) \begin{bmatrix} A_i & B_i \end{bmatrix}. \quad (4.9)$$

with $\alpha(t)$ belonging to the unit simplex

$$\Gamma_N = \left\{ \beta \in \mathbb{R}^N : \sum_{i=1}^N \beta_i = 1, \beta_i \geq 0, i = 1, \dots, N \right\}. \quad (4.10)$$

The states are assumed to be available at sampling instants t_k , such that $t \in \cup_{k \in \mathbb{N}} [t_k, t_{k+1}) = [0, +\infty)$, with $t_{k+1} - t_k = \eta_k \leq \mathcal{T}$, where η_k may be aperiodic but is bounded by the maximum allowable upper bound $\mathcal{T}$. Moreover, the control signal at $t = t_k$ is kept constant by a zero-order holder until the next sample. The sampled-data state feedback control law (Fridman, 2010; Seuret, 2012) is considered:

$$u(t) = Kx(t_k) = Kx(t - \tau(t)), \quad (4.11)$$

where $K \in \mathbb{R}^{n_u \times n}$ is a robust control gain matrix and, $\forall t \in [t_k, t_{k+1})$, $\tau(t) = t - t_k \in [0, \mathcal{T}]$ with $\dot{\tau}(t) = 1$. This formulation yields the closed-loop dynamics:

$$\dot{x}(t) = A(\alpha(t))x(t) + B(\alpha(t))Kx(t - \tau(t)). \quad (4.12)$$

In this application, the robust control gain K is assumed to be known. To establish the closed-loop stability conditions of the sampled-data system, summarized in the following theorem, a

path similar to that of [Seuret \(2012\)](#), is followed, but with a slightly modified looped-functional given by:

$$\begin{aligned} V(x(t)) &= x^\top(t)Px(t), \\ \mathcal{V}_0(x(t), \tau(t)) &= (\eta_k - \tau(t)) \left[(x(t) - x(t - \tau(t)))^\top (S(x(t) - x(t - \tau(t))) + 2Qx(t - \tau(t))) \right. \\ &\quad \left. + \tau(t)\zeta^\top U\zeta + \int_{t-\tau(t)}^t \dot{x}^\top(s)R\dot{x}(s)ds \right], \end{aligned} \quad (4.13)$$

with $\zeta = \begin{bmatrix} x^\top(t) & x^\top(t - \tau(t)) \end{bmatrix}^\top$. The inclusion of the term ζ is proposed in this work to obtain a looped-functional whose derivative is a second-order function in $\tau(t)$. Note that (4.13) satisfies the looped conditions in Theorem 2.3.

Theorem 4.4. *For given $\mathcal{T} > 0$ and $K \in \mathbb{R}^{n_u \times n}$, the closed-loop LPV sampled-data system (4.12) is asymptotically stable, if there exist matrices $\mathbf{0} < R \in \mathbb{S}^n$, $\mathbf{0} < P \in \mathbb{S}^n$, $S \in \mathbb{S}^n$, $U \in \mathbb{S}^{2n}$, $Q \in \mathbb{R}^{n \times n}$, $L \in \mathbb{R}^{3n \times n}$, and $X \in \mathbb{R}^{3n \times n}$ satisfying:*

$$\tau^2(t)\Pi_2 + \tau(t)\Pi_1 + \Pi_0(\alpha(t)) < \mathbf{0}, \quad \forall \tau(t) \in [0, \mathcal{T}] \quad (4.14)$$

with

$$\begin{aligned} \Pi_2 &= -\text{He}\{\bar{E}_1^\top U \bar{E}_2\}, \\ \Pi_1 &= \text{He}\{-E_3^\top (QE_2 + SE_{12})\} - \mathcal{T}\Pi_2 - 2\bar{E}_1^\top U \bar{E}_1 - E_3^\top RE_3 - LR^{-1}L^\top, \\ \Pi_0(\alpha(t)) &= \text{He}\{E_1^\top PE_3 - E_{12}^\top QE_2 - LE_{12} + \mathcal{T}E_3^\top (QE_2 + SE_{12})\} \\ &\quad + \mathcal{T}(E_3^\top RE_3 + \bar{E}_1^\top U \bar{E}_1) - E_{12}^\top SE_{12} + \text{He}\{X\mathcal{G}(\alpha(t))\}, \end{aligned}$$

$$\mathcal{G}(\alpha(t)) = \begin{bmatrix} A(\alpha(t)) & B(\alpha(t))K & -I \end{bmatrix}, \quad E_1 = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad E_2 = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \end{bmatrix}, \quad E_3 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}, \quad \bar{E}_1^\top = \begin{bmatrix} E_1^\top & E_2^\top \end{bmatrix}, \quad \bar{E}_2^\top = \begin{bmatrix} E_3^\top & \mathbf{0} \end{bmatrix}, \quad \text{and } E_{12} = E_1 - E_2.$$

The maximum allowed upper bound $\mathcal{T}$ can be obtained by linear search, i.e., by increasing $\mathcal{T}$ until the quadratic inequality (4.14) remains feasible.

Proof: Assume that $\mathbf{0} < P \in \mathbb{S}^n$, then $V(x(t)) > 0$ and (2.31) is satisfied. Since $\mathcal{V}_0(x(t), 0) = \mathcal{V}_0(x(t), \mathcal{T}) = 0$, (2.32) is also verified. Therefore, the closed-loop system (4.12) is asymptotically stable if (2.33) is satisfied. The respective time-derivative $\dot{\mathcal{W}}(x(t), \tau(t))$ is given by:

$$\begin{aligned} \dot{\mathcal{W}}(x(t), \tau(t)) &= 2\dot{x}^\top(t)Px(t) - (x(t) - x(t - \tau(t)))^\top S(x(t) - x(t - \tau(t))) \\ &\quad + (\eta_k - \tau(t))2\dot{x}^\top(t)S(x(t) - x(t - \tau(t))) - 2(x(t) - x(t - \tau(t)))^\top Qx(t - \tau(t)) \\ &\quad + (\eta_k - \tau(t))2\dot{x}^\top(t)Qx(t - \tau(t)) - \tau(t)\zeta^\top U\zeta + (\eta_k - \tau(t))\zeta^\top U\zeta \\ &\quad + (\eta_k - \tau(t))\tau(t)2\dot{\zeta}^\top U\zeta + (\eta_k - \tau(t))\dot{x}^\top(t)R\dot{x}(t) - \int_{t-\tau(t)}^t \dot{x}^\top(s)R\dot{x}(s)ds < 0, \end{aligned}$$

with $\dot{\zeta} = \begin{bmatrix} \dot{x}^\top(t) & \mathbf{0} \end{bmatrix}^\top$. Applying Jensen's inequality (see Eq. (9) of [Briat \(2011\)](#)), the last term of $\dot{\mathcal{W}}(x(t), \tau(t))$ can be upper bounded with

$$- \int_{t-\tau(t)}^t \dot{x}^\top(s)R\dot{x}(s)ds \leq -2\xi^\top L(x(t) - x(t - \tau(t))) + \tau(t)\xi^\top LR^{-1}L^\top \xi,$$

with $\mathbf{0} < R \in \mathbb{S}^n$, $L \in \mathbb{R}^{3n \times n}$, and the augmented state vector $\xi = \begin{bmatrix} x^\top(t) & x^\top(t - \tau(t)) & \dot{x}^\top(t) \end{bmatrix}^\top$. Thus, it follows that

$$\dot{W}(x(t), \tau(t)) \leq \xi^\top \left(\tau^2(t) \Pi_2 + \tau(t) \Pi_1 + \Pi_0(\alpha(t)) \right) \xi.$$

From (4.12), it yields $\mathcal{G}(\alpha(t))\xi(t) = 0$. Applying Finsler's Lemma (de Oliveira and Skelton, 2001), it leads to (4.14) with the introduction of the slack decision matrix $X \in \mathbb{R}^{3n \times n}$, considering the upper bound of η_k as $\mathcal{T}$. ■

Remark 4.5. Condition (4.14) is a quadratic constraint (4.1) with a time-varying parameter-dependent $\Pi_0(\alpha(t))$. To address the conditions outlined in Theorem 4.4, the conditions of Lemmas 4.1 and 4.2, or Theorems 4.1, 4.2, and 4.3, can be applied to all vertices of

$$(A(\alpha(t), B(\alpha(t))) \in \text{Co}\{(A_i, B_i), \forall i = 1, \dots, N\},$$

with additional Schur complements used to handle the term $LR^{-1}L^\top$. See also Remark 4.1 and Appendix A for more details on how to apply Theorem 4.2 to solve (4.14).

Remark 4.6. The looped-functional (4.13) is standard, lacking additional terms such as multiple integrals as often seen in previous studies to reduce conservatism (e.g., Lee and Park (2017); Lee et al. (2018)). The proof of Theorem 4.4 is also simple, employing Jensen's inequality (see Theorem 4.1 in Briat (2011)), while other studies typically use tighter bounding techniques such as Wirtinger (Seuret and Gouaisbaut, 2013), Bessel-Legendre inequalities (Lee and Park, 2017; Gao et al., 2020), or combinations, as relaxation techniques. In fact, in most previous related studies that provide new negative definiteness conditions for (4.1), comparisons are made with other studies employing different relaxation techniques (see, e.g., Kim (2011); Yang and Zhang (2014); Kim (2016); Yang et al. (2019); Chen et al. (2019); Zhang et al. (2020a, 2022b); Chen et al. (2022b); He et al. (2023); Liu et al. (2023)). Hence, in these previous studies, it is difficult to distinguish conservative improvements due only to the proposed negative definiteness conditions for (4.1). In this chapter, within the same framework used for the stability analysis of LPV sampled-data systems, the objective is to fairly compare the different approaches presented above to address (4.1). The numerical results are presented in the next section.

4.4 Illustrative Examples

In this section, the example of a quadratic scalar inequality (4.1) is first considered, followed by another example dealing with the stability analysis of an LPV sampled-data control plant. LMI conditions are solved using YALMIP (Löfberg, 2004) and Mosek solver (Andersen and Andersen, 2000). All numerical tests are performed on an MSI GS60 2PL laptop equipped with a 2.5GHz Intel Core i7-4710HQ processor and 12GB of RAM, running Windows 10.

4.4.1 Numerical Example of a Scalar Quadratic Inequality

Consider the scalar quadratic inequality:

$$\mathcal{P}(\tau) = \tau^2 10a + \tau 10 + b - a < 0, \quad \tau \in [0, 1], \quad (4.15)$$

where a and b are real parameters used to evaluate and compare the feasibility fields of the different negative definiteness conditions. Since (4.15) is a scalar, it can be asserted that, for $\tau \in [0, 1]$, $\mathcal{P}(\tau) < 0$ if $(a, b) \in \mathcal{S}$, with $\mathcal{S}$ defined by:

$$\mathcal{S} = \left\{ (a, b) \in \mathbb{R}^2 \left| \begin{array}{l} \mathcal{P}(0) = b - a < 0, \\ \mathcal{P}(1) = 9a + b + 10 < 0, \\ b - a - \frac{5}{2a} < 0, \text{ if } -\frac{1}{2a} \in [0, 1]. \end{array} \right. \right\} \quad (4.16)$$

This characterization of the domain $\mathcal{S}$ where (4.15) holds is used to compare the conservatism, in terms of feasibility fields, of Lemmas 4.1-4.3 and Theorems 4.1-4.3.

First, consider the special case where $a = -0.66$ and $b = -4.46$. Table 4.2 shows feasible solutions for some N_p under partitioning conditions in Lemma 4.2, Theorem 4.1, and Theorem 4.2 (with fixed d), while $N_p + 1$ does not yield solutions ($\emptyset$). This highlights the non-MC behavior in terms of N_p of these conditions, and so confirms Fact 4.1. The MC behaviors of Theorem 4.2 (with respect to d for fixed N_p) and Theorem 4.3 (with respect to $\mathcal{N}$) are demonstrated in the sequel. Then, for $a \in [-16, 1]$, $b \in [-16, -3]$, and without partition ($N_p = 1$), a comparison Table 4.2: Feasibility of the different negative definiteness conditions (with respect to N_p , d or $\mathcal{N}$) with (4.15), $a = -0.66$ and $b = -4.46$.

Methods / N_p (or $\mathcal{N}$) :	1	2	3	4	5	6
Lemma 4.2, <i>i</i>) Chen et al. (2019)	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
Lemma 4.2, <i>ii</i>) He et al. (2023)	$\emptyset$	$\emptyset$	$\emptyset$	Feasible	$\emptyset$	$\emptyset$
Lemma 4.2, <i>iii</i>) Liu et al. (2023)	$\emptyset$	$\emptyset$	$\emptyset$	Feasible	$\emptyset$	$\emptyset$
Theorem 4.1	$\emptyset$	Feasible	$\emptyset$	Feasible	$\emptyset$	Feasible
Theorem 4.2 , $d = 0$	$\emptyset$	$\emptyset$	$\emptyset$	Feasible	$\emptyset$	$\emptyset$
Theorem 4.2 , $d = 1$	$\emptyset$	$\emptyset$	$\emptyset$	Feasible	$\emptyset$	$\emptyset$
Theorem 4.2 , $d = 2$	$\emptyset$	$\emptyset$	$\emptyset$	Feasible	$\emptyset$	$\emptyset$
Theorem 4.2 , $d = 3$	$\emptyset$	$\emptyset$	$\emptyset$	Feasible	Feasible	Feasible
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Theorem 4.2 , $d = 9$	$\emptyset$	$\emptyset$	$\emptyset$	Feasible	Feasible	Feasible
Theorem 4.2 , $d = 10$	$\emptyset$	$\emptyset$	Feasible	Feasible	Feasible	Feasible
Theorem 4.3	$\emptyset$	Feasible	Feasible	Feasible	Feasible	Feasible

of the feasibility fields obtained from the different negative definiteness conditions is shown in Figure 4.3. Lemma 4.1 provides the most conservative results, while Theorem 4.2 (with $d = 0$ and $N_p = 1$) matches Theorem 4.3 (with $\mathcal{N} = 0$), and the less conservative geometric conditions in Lemmas 4.1 and 4.2. Furthermore, Theorem 4.1 encompasses the latter results, while Theorem 4.2 (with $d = 10$ and $N_p = 1$) provides a wider feasibility field. Hence, the present proposals consistently outperform or equalize the results obtained from Lemmas 4.1 and 4.2. Note that Lemma 4.3 allows recovery of $\mathcal{S}$ with this simple example. However, this may not always be true, due to Fact 4.2.

The last test compares the feasibility fields of Theorems 4.1-4.3 (for various N_p , d , and $\mathcal{N}$) with the most effective previous partitioning approaches. The results are shown in Figure 4.4. In Figure 4.4 (a), as N_p increases, Lemma 4.2 *ii*), *iii*), and Theorem 4.2 (with $d = 0$), exhibit

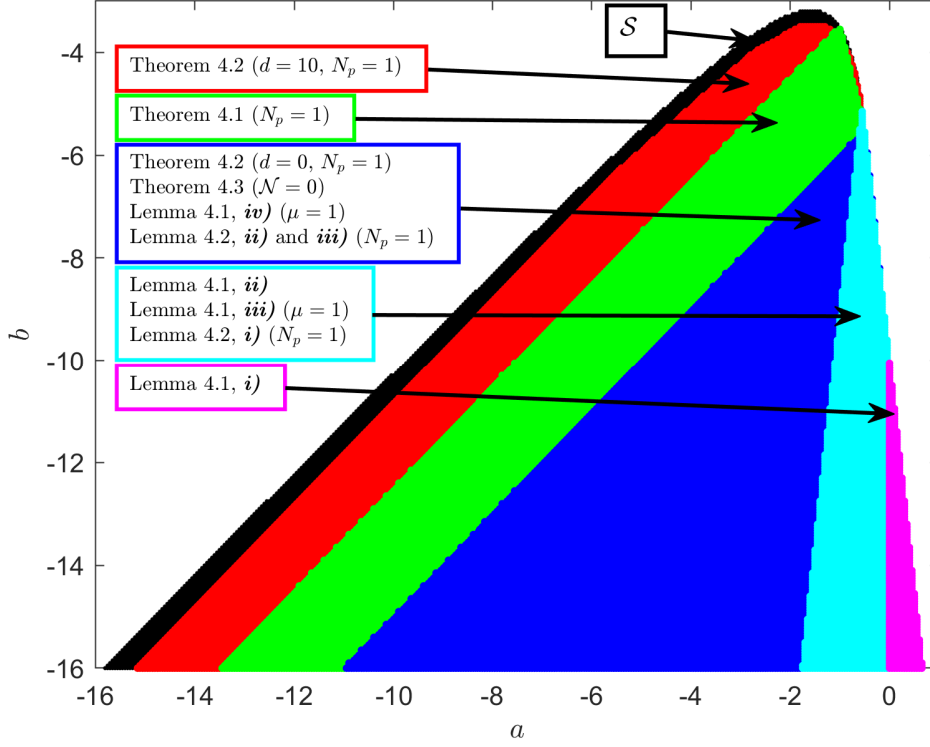


Figure 4.3: Comparison of the feasibility fields obtained from the different approaches with no partitions ($N_p = 1$ or $\mathcal{N} = 0$).

identical results. Notably, expanding N_p does not cover all areas consistently, as $N_p + 1$ misses certain solutions found at lower N_p values, again confirming Fact 4.1. In Figure 4.4 (b), Theorem 4.1 also demonstrates non-MC behavior, but the coverage area for a given N_p consistently exceeds that of Figure 4.4 (a), indicating a reduction in conservatism. In contrast, Theorem 4.2 provides MC conditions by increasing d , as shown in Figure 4.4 (c), (d), and (e) for fixed $N_p = 1$, 2, and 5, respectively, where the areas for $d + 1$ always include those for d . Then, Theorem 4.2 with $N_p = 5$ and $d = 4$ closely approximates $\mathcal{S}$. Figure 4.4(f) shows results from Theorem 4.3, increasing de Casteljau's iterations from $\mathcal{N} = 1$ to 5 (with tighter approximations achieved for large $\mathcal{N}$), outperforming all the other tested results in terms of coverage area. Furthermore, the inclusion of the results for $\mathcal{N} + 1$ in $\mathcal{N}$ confirms the MC behavior of Theorem 4.3.

This first example demonstrates the effectiveness and advantages of the proposed conditions, highlighting Fact 4.1 alongside Lemma 4.2, Theorem 4.1, and Theorem 4.2 (with respect to N_p for fixed d). Additionally, it reveals the MC behavior of Theorem 4.2 (with respect to d for fixed N_p) and Theorem 4.3, addressing the drawback of non-MC conditions.

4.4.2 Benchmark of a Sampled-Data Control System

In this example, the stability of a LPV sampled-data closed-loop system (4.12) is investigated for the largest allowable upper bound $\mathcal{T}$ of the aperiodic sampling intervals, with vertices given by:

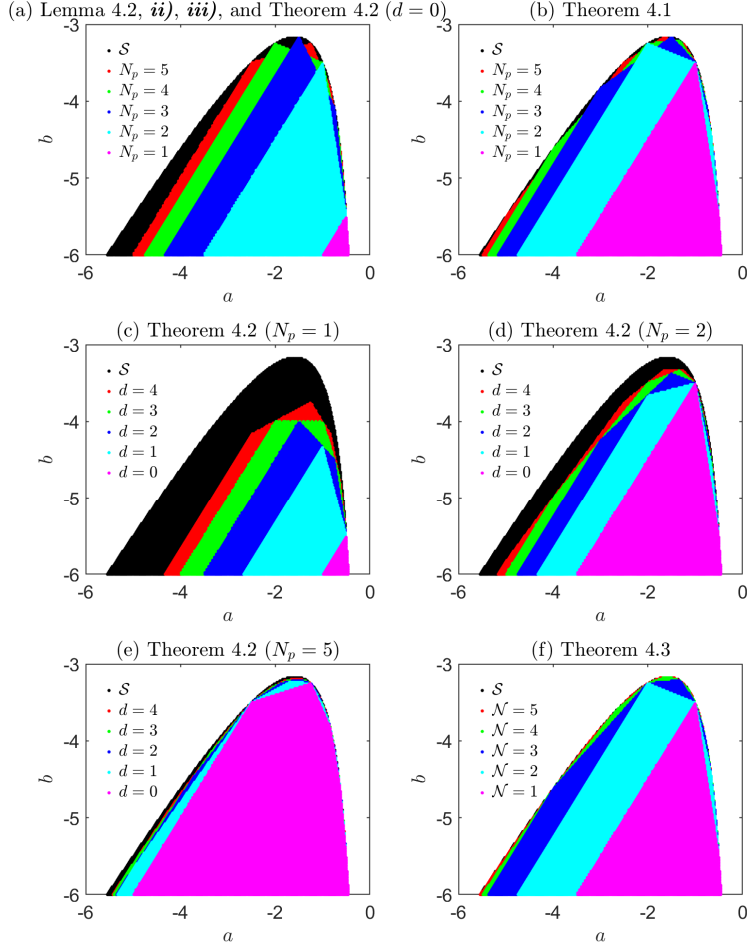


Figure 4.4: Comparison of the feasibility fields satisfying (4.15), obtained from Lemmas 4.2 and Theorems 4.1-4.3, for various N_p , d , or $\mathcal{N}$.

$$A_i = \begin{bmatrix} \gamma_i & 0 \\ 0 & \theta_i \end{bmatrix}, B_i K = \begin{bmatrix} -1 & 0 \\ -1 & \lambda_i \end{bmatrix}$$

where $\gamma_i = -1.8$ for $i \in \{1, 5, 2, 6\}$, $\gamma_i = -2.2$ for $i \in \{3, 7, 4, 8\}$, $\theta_i = -0.8$ for $i \in \{1, 5, 3, 7\}$, $\theta_i = -1$ for $i \in \{2, 6, 4, 8\}$, $\lambda_i = -0.8$ for $i \in \{1, 2, 3, 4\}$ and $\lambda_i = -1.2$ for $i \in \{5, 6, 7, 8\}$.

According to Remark 4.5, the quadratic conditions of Theorem 4.4 are addressed using Lemmas 4.1-4.3 and Theorems 4.1-4.3. Table 4.3 shows the maximum allowable sampling time $\mathcal{T}$ achieved with various values of N_p , d , or $\mathcal{N}$. The highest value obtained is $\mathcal{T} = 1.950$. Among geometric conditions in Lemmas 4.1-4.3, only those in Lemma 4.2, ii) and iii), reach this maximum with $N_p = 29$ partitions (indicating conservatism in the other conditions). Moreover, despite the fact that Lemma 4.3 is reputed to be necessary and sufficient when Π_0 , Π_1 , and Π_2 are parameter independent, it is only sufficient for the uncertain parameter-dependent case studied in this example. This confirms Fact 4.2 since the achieved value $\mathcal{T} = 1.926$ s is smaller than the largest one obtained with Lemma 4.2, ii) or iii) or Theorems 4.1-4.3. In particular, Theorems 4.1-4.3 also reach $\mathcal{T} = 1.950$ s but with fewer (or equal) partitions (or de Casteljau's

iterations) than Lemma 4.2, *ii*) and *iii*).

Table 4.3: Maximum allowable sampling interval in seconds ($\mathcal{T}$) and solver time (t_{sol}), obtained from Lemmas 4.2-4.3 and Theorems 4.1-4.3 (with respect to d , N_p or $\mathcal{N}$).

Methods	$\mathcal{T} \mid N_p \text{ or } \mathcal{N} \mid t_{sol}(s)$		
Lemma 4.1, <i>ii</i>) (Kim, 2016; Yang et al., 2019)	1.865	-	0.061
Lemma 4.1, <i>iii</i>) (Zhang et al., 2020a), $\mu = 0$	1.865	-	0.079
Lemma 4.1, <i>iii</i>) (Zhang et al., 2020a), $\mu = 1$	1.851	-	0.048
Lemma 4.1, <i>iii</i>) (Zhang et al., 2020a), $\mu = 0.15$	1.881	-	0.067
Lemma 4.1, <i>iv</i>) (Liu et al., 2023), $\mu = 0$	1.867	-	0.059
Lemma 4.1, <i>iv</i>) (Liu et al., 2023), $\mu = 1$	1.867	-	0.056
Lemma 4.1, <i>iv</i>) (Liu et al., 2023), $\mu = 0.2$	1.888	-	0.061
Lemma 4.2, <i>i</i>) (Chen et al., 2019)	1.851	1	0.043
Lemma 4.2, <i>ii</i>) (He et al., 2023)	1.867	1	0.045
Lemma 4.2, <i>iii</i>) (Liu et al., 2023)	1.867	1	0.047
Lemma 4.3 (Park and Park, 2020; Kim, 2021)	1.926	-	0.038
Theorem 4.1	1.885	1	0.057
Theorem 4.2 , $d = 0$	1.867	1	0.042
Theorem 4.2 , $d = 1$	1.870	1	0.061
Theorem 4.2 , $d = 5$	1.906	1	0.145
Theorem 4.2 , $d = 10$	1.925	1	0.286
Theorem 4.3	1.885	1	0.086

It is fair to say that the conditions in Theorems 4.1-4.3 are more computationally demanding, as revealed by comparing the solver time t_{sol} required for each result that reaches the maximum $\mathcal{T} = 1.950$ (with minimal N_p , polynomial degree d , or de Casteljau iterations $\mathcal{N}$). However, it is important to recall that the conditions of Lemma 4.2 and Theorem 4.1 are non-MC, so reaching the maximum does not guarantee optimality. On the contrary, Theorems 4.2 and 4.3 being MC, ensure that $\mathcal{T} = 1.950$ is the optimal solution for the quadratic constraints in Theorem 4.4. Furthermore, Theorems 4.2 and 4.3 offer more flexibility in adjusting d , N_p , or $\mathcal{N}$, to balance conservatism improvements and computational complexity. Hence, it can be claimed that the increase in computational complexity is the price to pay for less conservative MC conditions (unexplored in the previous literature) to provide guaranteed optimal solutions in terms of negative definiteness conditions, which is an important feature in various control problems.

4.5 Final Considerations

This chapter introduces new approaches to tackle the negative definiteness of quadratic functions, crucial in stability analysis and control design for time-varying delayed systems. These are based on rewriting the quadratic functions as homogeneous polynomials or Bézier curves. First, less conservative non-MC conditions are suggested using Young's inequality, followed by MC conditions via Pólya's relaxations or de Casteljau's algorithm. Unlike prior non-MC partitioning methods, the proposed less conservative MC conditions, based on Pólya's Theorem or de Casteljau's algorithm, ensure optimal solutions in terms of negative definiteness conditions

for sufficiently large d (Pólya's degree) or $\mathcal{N}$ (number of de Casteljau's iterations). This important feature for control problems is illustrated first through a scalar quadratic function example and then applied to an LPV sampled-data control benchmark. Both examples demonstrate the advantages over the most effective negative definiteness conditions from previous literature.

Mitigating Parameter Mismatch in LPV Sampled-Data Controller Design

Michael Lindsay-Hogg: Well, now, what about the roof tomorrow? Do you wanna collectively do it or not?

Paul McCartney: Let's... Let's decide on that, sort of, a bit later. Let's us keep off that. We'll do the numbers, you know? We're the band.

George Harrison: You know, whatever. I'll do it if we've got to go on the roof... but I don't wanna go on the roof. Of course I don't wannna go on the roof.

Ringo Starr: I would like to go on the roof.

Paul McCartney: You would like to?

John Lennon: Yes, I'd like to go on the roof.

The Beatles: Get Back (2021)

dir. PETER JACKSON, from the rehearsal recorded by Michael Lindsay-Hogg on January 29, 1969, the day before the rooftop concert, the last public performance of their career.

This chapter presents a refined approach to cope with the problem of parameter mismatch in the design conditions of sampled-data controllers for LPV systems, which occurs between consecutive sampling instants. Using the input-delay approach, a novel strategy is proposed that applies the mean value theorem while explicitly incorporating the delay into the stability conditions. This method avoids the spread of mismatch terms across all parameter-dependent matrices, yielding less conservative controller synthesis conditions and enabling larger sampling intervals than existing techniques. Despite its simplicity, the presented approach demonstrates superior performance relative to conditions in the literature, as shown by a numerical example. Furthermore, this method is explored jointly with the negative definiteness of quadratic polynomials in the sampled-data controller synthesis case.

5.1 Parameter-Dependent Stabilization of LPV Sampled-Data Systems via Linear Polynomial Condition

Consider a continuous-time LPV system given by:

$$\dot{x}(t) = A(\alpha(t))x(t) + B(\alpha(t))u(t), \quad (5.1)$$

with $x(t) \in \mathbb{R}^n$ denoting the state vector and $u(t) \in \mathbb{R}^{n_u}$ representing the control signal vector. The vector of the time-varying parameter is represented by $\alpha(t) \in \Gamma_N$, where

$$\Gamma_N = \{\delta \in \mathbb{R}^N : \sum_{i=1}^N \delta_i = 1, \delta_i \geq 0, i = 1, \dots, N\}.$$

It is assumed that, $\forall t \geq 0$, $\alpha_i(t) \in \mathcal{C}^1$ (i.e., α_i are continuous function as well as their derivatives) and $|\dot{\alpha}_i(t)| \leq \phi_i$, for $i = 1, \dots, N$. The matrices $A(\alpha(t)) \in \mathbb{R}^{n \times n}$ and $B(\alpha(t)) \in \mathbb{R}^{n \times n_u}$ are polytopic with N vertices described by

$$\begin{bmatrix} A(\alpha(t)) & B(\alpha(t)) \end{bmatrix} = \sum_{i=1}^N \alpha_i(t) \begin{bmatrix} A_i & B_i \end{bmatrix}. \quad (5.2)$$

Consider a sampled-data control law depending both on the state and the parameter of the system available at asynchronous sampling instants, t_k , given by:

$$u(t) = K(\alpha(t_k))x(t_k), \quad \forall t \in [t_k, t_{k+1}) \quad (5.3)$$

where t_k belongs to an increasing sequence of positive scalar values of sampling instants t_k , such that $\cup_{k \in \mathbb{N}} [t_k, t_{k+1}) = [0, +\infty)$, and $K(\alpha(t_k))$ follows similar structure as (5.2). Additionally, the difference between two successive sampling instants is given by $\eta_k = t_{k+1} - t_k$. Such a difference may change at each sampling interval, however, always bounded by two positive scalars $\mathcal{T}_1 \leq \mathcal{T}_2$, such that $\forall k \in \mathbb{N}$ it verifies $0 < \mathcal{T}_1 \leq \eta_k \leq \mathcal{T}_2$.

Replacing (5.3) into (5.1), it yields

$$\dot{x}(t) = A(\alpha(t))x(t) + B(\alpha(t))K(\alpha(t_k))x(t_k), \quad (5.4)$$

where the time difference between the parameter of the system, $\alpha(t)$, and the parameter of the controller, $\alpha(t_k)$, is notable. Such a difference causes mismatch between matrices ($A(\alpha(t))$, $B(\alpha(t))$, and $K(\alpha(t_k))$) and states ($x(t)$ and $x(t_k)$). Using the time-delay approach (Fridman, 2010), the sampling instant can be written in terms of $\tau(t) \in [0, \eta_k]$ such that $\dot{\tau}(t) = 1$, as $t_k = t - \tau(t)$, $\forall t \in [t_k, t_{k+1})$. Thus, the dynamic of the sampled-data system is given as

$$\dot{x}(t) = A(\alpha(t))x(t) + B(\alpha(t))K(\alpha(t - \tau(t)))x(t - \tau(t)). \quad (5.5)$$

As noted by Gomes da Silva Jr. et al. (2015), sampled-data LPV systems face a challenge in accurately modeling the evolution of time-varying parameters between sampling instants. While both $\alpha(t)$ and $\alpha(t - \tau(t))$ lie in the same polytopic structure, assuming no evolution of these parameters within the sampling interval (Tan et al., 2002) can lead to instability in

practical applications, as this assumption does not hold in real-world systems. On the other hand, the approach of [Shi and Su \(2014\)](#), which considers independent variation of $\alpha(t)$ and $\alpha(t - \tau(t))$, mitigates instability but results in overly conservative design conditions. To address this, [Gomes da Silva Jr. et al. \(2015\)](#) introduced a refined approach where the parameters are modeled using two distinct polytopes: a sampled one, lying in the limits of the parameter itself, and a continuous-time-varying one that is bounded by the parameter rate of variation in the intersample interval given by $\pm\phi_i\mathcal{T}_2$, with ϕ_i denoting the maximum variation rate of the i -th element of the parameter vector $\alpha(t)$. These bounds had been adopted as a standard for the sampled-data LPV framework in many previous studies, see e.g., ([Gomes da Silva Jr. et al., 2018](#); [Hooshmandi et al., 2018a,b, 2020](#); [Jung et al., 2021](#)).

However, a limitation of this approach lies in its inherent conservatism. Indeed, by decoupling the estimates of $\alpha(t)$ and $\alpha(t_k)$, some unrealistic combinations of these parameters are included, leading to overly cautious controller designs. For instance, this occurs when $\alpha_1(t_k) = 1$ or 0, while the polytope representation of the intersample suggests that the parameters can go beyond their limits, which is unrealistic. This chapter aims to refine this modeling further by introducing improved mismatched parameter representation techniques that reduce this conservatism, thereby ameliorating the achievable performance and stability margins.

The objective of this chapter is to propose a simple and new alternative based on the mean value theorem to rewrite the closed-loop equations explicitly depending on the input delay $\tau(t)$. This enables the incorporation of $\tau(t)$ into the proposed stabilization conditions, thereby mitigating the effects of the modeling mismatch between the parameters of the system and the controller, which leads to less conservative controller design conditions. The approach yields, for instance, larger estimates of $\mathcal{T}_2$ for a given $\mathcal{T}_1$ in asynchronous samplings than other recent findings in the literature. Thus, the main contribution of this chapter is to provide a solution to the following problem.

Problem 5.1. *Given the continuous-time LPV system represented by (5.1), and a sampled-data control law (5.3), design the parameter-dependent control gain matrix $K(\alpha(t_k))$ such that the maximum allowable sampling interval is maximized and the closed-loop is asymptotically stable for all $\eta_k \in [\mathcal{T}_1, \mathcal{T}_2]$.*

5.1.1 Looped-Functionals and The Mean Value Approach

To guarantee the stability of the closed-loop system (5.5), the following lemma proposed in [Gomes da Silva Jr. et al. \(2018, Lemma 1\)](#) based on [Seuret \(2012\)](#) is considered:

Lemma 5.1 ([Gomes da Silva Jr. et al. \(2018\)](#)). *Consider positive scalars κ_1 , κ_2 , and κ_3 , a parameter-dependent function $V : \mathbb{K} \times \Gamma_N \rightarrow \mathbb{R}_+$ satisfying*

$$\kappa_1\|x\|^2 \leq V(x, \alpha) \leq \kappa_2\|x\|^2, \quad (5.6)$$

and a continuous and differentiable parameter-dependent function $\mathcal{V}_0 : \mathbb{K} \times \Gamma_N \times [0, \eta_k] \times [\mathcal{T}_1, \mathcal{T}_2] \rightarrow \mathbb{R}$ satisfying

$$\mathcal{V}_0(x, \alpha, 0, \eta_k) = \mathcal{V}_0(x, \alpha, \eta_k, \eta_k), \quad (5.7)$$

for all $x \in \mathbb{K}$, $\alpha \in \Gamma_N$, $\tau \in [0, \eta_k]$, and $\eta_k \in [\mathcal{T}_1, \mathcal{T}_2]$.

Let $\mathcal{W}(x, \alpha, \tau, \eta_k) = V(x, \alpha) + \mathcal{V}_0(x, \alpha, \tau, \eta_k)$. If the inequality

$$\dot{\mathcal{W}}(x, \alpha, \tau, \eta_k) = \frac{d}{dt}[V(x, \alpha) + \mathcal{V}_0(x, \alpha, \tau, \eta_k)] < -\kappa_3 \|x\|^2 \quad (5.8)$$

is satisfied, then:

1. The increment of the Lyapunov function $V(x(t), \alpha(t))$ between two consecutive sampling instants is strictly negative, i.e., it satisfies

$$\Delta V(x(t_k), \alpha(t_k)) = V(x(t_{k+1}), \alpha(t_{k+1})) - V(x(t_k), \alpha(t_k)) < 0;$$

2. The sampled-data closed-loop system (5.4) with $\eta_k \in [\mathcal{T}_1, \mathcal{T}_2]$ is asymptotically stable for all $(k, \alpha(t), \tau(t), \eta_k) \in \mathbb{N} \times \Gamma_N \times [0, \eta_k] \times [\mathcal{T}_1, \mathcal{T}_2]$.

A key result used in this chapter is the well-known mean value theorem, recalled in the following.

Theorem 5.1. Let $f : [a, b] \rightarrow \mathbb{R}$ be a continuous function on $[a, b]$. Then, there exists $c \in [a, b]$ such that $\int_a^b f(s)ds = (b - a)f(c)$.

From Theorem 5.1, $\forall i = 1, \dots, N$, $\exists c_i(t) \in [0, \tau(t)]$ such that

$$\alpha_i(t) - \alpha_i(t - \tau(t)) = \int_{t-\tau(t)}^t \dot{\alpha}_i(s)ds = \tau(t)\dot{\alpha}_i(t - c_i(t)). \quad (5.9)$$

Therefore, $K(\alpha(t - \tau(t)))$ can be written as:

$$K(\alpha(t - \tau(t))) = K(\alpha(t)) - \sum_{i=1}^N \int_{t-\tau(t)}^t \dot{\alpha}_i(s)ds K_i = K(\alpha(t)) - \tau(t)\dot{K}(\alpha(t - c(t))), \quad (5.10)$$

where $\dot{K}(\alpha(t - c(t))) = \sum_{i=1}^N \dot{\alpha}_i(t - c_i(t))K_i$, leading to rewrite the control law (5.3) as:

$$u(t) = (K(\alpha(t)) - \tau(t)\dot{K}(\alpha(t - c(t))))x(t - \tau(t)), \quad (5.11)$$

and so the closed-loop dynamics (5.5) is equivalent to

$$\dot{x}(t) = A(\alpha(t))x(t) + B(\alpha(t)) \left(K(\alpha(t)) - \tau(t)\dot{K}(\alpha(t - c(t))) \right) x(t - \tau(t)). \quad (5.12)$$

Therefore, the control law (5.11) is shown to be equivalent to (5.3). This equivalence is established by utilizing the mean value theorem to reformulate the control law. This reformulation is essential to derive the stabilization conditions, proposed in the next section, and to compute the corresponding matrix gains while ensuring that the sampled-data control law remains implemented using the standard parameter-dependent controller presented in (5.3).

The next section employs the looped-functional approach introduced in Seuret (2012) to design the sampled-data control law (5.3), ensuring stability of the closed-loop system (5.12), or equivalently (5.5), provided that some parameterized LMIs, affine in $\tau(t)$, are satisfied.

Remark 5.1. Unlike prior studies on LPV sampled-data systems, where the mismatch is handled by assuming $|\alpha_i(t) - \alpha_i(t_k)| \leq \phi_i \mathcal{T}_2$ (see, e.g., [Gomes da Silva Jr. et al. \(2015\)](#); [Hooshmandi et al. \(2018a,b, 2020\)](#); [Jung et al. \(2021\)](#)), the reformulation of the closed-loop dynamics as in (5.12) enables putting the term $\dot{K}(\alpha(t - c(t)))$ only into the first order term of the parametrized LMIs affine in $\tau(t)$, as shown in the next section with the main results. In this context, when $\tau(t) = 0$, the mismatch vanishes, reducing conservatism compared to previous approaches, which still need to address $\dot{K}(\alpha(t - c(t)))$ in the lower order of the parametrized LMIs affine in $\tau(t)$.

5.1.2 Stabilization Conditions

The following theorem provides a solution to Problem 5.1.

Theorem 5.2. If there exists matrices $\mathbf{0} < \tilde{P}(\alpha(t)) \in \mathbb{S}^n$, $\mathbf{0} < \tilde{R} \in \mathbb{S}^n$, $\tilde{S}(\alpha(t)) \in \mathbb{S}^n$, $\tilde{Q}(\alpha(t)) \in \mathbb{R}^{n \times n}$, $\tilde{U} \in \mathbb{S}^n$, $\tilde{L}(\alpha(t)) \in \mathbb{R}^{3n \times n}$, $\tilde{K}(\alpha(t)) \in \mathbb{R}^{n_u \times n}$, $\tilde{X} \in \mathbb{R}^{n \times n}$, and a scalar $\epsilon > 0$ satisfying the following conditions

$$\tilde{\Pi}_0 + \mathcal{T}_i(\tilde{\Pi}_1 + \tilde{\Pi}_2) < \mathbf{0}, \quad (5.13)$$

$$\begin{bmatrix} \tilde{\Pi}_0 - \mathcal{T}_i(\tilde{\Pi}_2 - \tilde{\Pi}_3) & \mathcal{T}_i \tilde{L}_\alpha \\ \star & -\mathcal{T}_i \tilde{R} \end{bmatrix} < \mathbf{0}, \quad (5.14)$$

$\forall i = 1, 2$, with

$$\begin{aligned} \tilde{\Pi}_0 &= \mathbf{He} \left\{ E_1^\top \tilde{P}(\alpha(t)) E_3 - E_{12}^\top \tilde{Q}(\alpha(t)) E_2 - \tilde{L}(\alpha(t)) E_{12} + (\epsilon E_1^\top + E_3^\top)(-A(\alpha(t)) \tilde{X} E_1 \right. \\ &\quad \left. - B(\alpha(t)) \tilde{K}(\alpha(t)) E_2 + \tilde{X} E_3) \right\} - E_{12}^\top \tilde{S}(\alpha(t)) E_{12} + E_1^\top \dot{\tilde{P}}(\alpha(t)) E_1, \\ \tilde{\Pi}_1 &= \mathbf{He} \left\{ E_3^\top (\tilde{Q}(\alpha(t)) E_2 + \tilde{S}(\alpha(t)) E_{12}) + E_{12}^\top \dot{\tilde{Q}}(\alpha(t)) E_2 \right\} + E_3^\top \tilde{R} E_3 + E_{12}^\top \dot{\tilde{S}}(\alpha(t)) E_{12}, \\ \tilde{\Pi}_2 &= E_2^\top \tilde{U} E_2, \\ \tilde{\Pi}_3 &= \mathbf{He} \left\{ \epsilon E_1^\top B(\alpha(t)) \dot{\tilde{K}}(\alpha(t - c(t))) E_2 + E_3^\top B(\alpha(t)) \dot{\tilde{K}}(\alpha(t - c(t))) E_2 \right\}, \end{aligned}$$

where $E_1 = [\mathbf{I} \ \mathbf{0} \ \mathbf{0}]$, $E_2 = [\mathbf{0} \ \mathbf{I} \ \mathbf{0}]$, $E_3 = [\mathbf{0} \ \mathbf{0} \ \mathbf{I}]$, $E_{12} = E_1 - E_2$. Then, system (5.1) is asymptotically stabilized by the sampled-data controller (5.3) where

$$K(\alpha(t_k)) = \tilde{K}(\alpha(t_k)) \tilde{X}^{-1},$$

for any sampling sequences satisfying $0 < \mathcal{T}_1 \leq \eta_k \leq \mathcal{T}_2$.

Proof: Consider a parameter-dependent looped-functional given by $\mathcal{W}(x(t), \alpha(t), \tau(t), \eta_k) = V(x(t), \alpha(t)) + \mathcal{V}_0(x(t), \alpha(t), \tau(t), \eta_k)$ where

$$V(x(t), \alpha(t)) = x^\top(t) P(\alpha(t)) x(t),$$

$$\begin{aligned} \mathcal{V}_0(x(t), \alpha(t), \tau(t), \eta_k) &= (\eta_k - \tau(t)) \left[(x(t) - x(t - \tau(t)))^\top \left(S(\alpha(t)) (x(t) - x(t - \tau(t))) \right. \right. \\ &\quad \left. \left. + 2Q(\alpha(t)) x(t - \tau(t)) \right) + \tau(t) x(t - \tau(t))^\top U x(t - \tau(t)) + \int_{t-\tau(t)}^t \dot{x}^\top(s) R \dot{x}(s) ds \right], \end{aligned}$$

with $\mathbf{0} < P(\alpha(t)) \in \mathbb{S}^n$, $\mathbf{0} < R \in \mathbb{S}^n$, $S(\alpha(t)) \in \mathbb{S}^n$, $U \in \mathbb{S}^n$, and $Q(\alpha(t)) \in \mathbb{R}^{n \times n}$, satisfying (5.6) and (5.7). The time derivative of this looped-functional candidate yields:

$$\begin{aligned}
 \dot{W}(x(t), \alpha(t), \tau(t), \eta_k) = & 2\dot{x}^\top(t)P(\alpha(t))x(t) + x^\top(t)\dot{P}(\alpha(t))x(t) \\
 & - (x(t) - x(t - \tau(t)))^\top S(\alpha(t))(x(t) - x(t - \tau(t))) \\
 & + (\eta_k - \tau(t))2\dot{x}^\top(t)S(\alpha(t))(x(t) - x(t - \tau(t))) \\
 & + (\eta_k - \tau(t))(x(t) - x(t - \tau(t)))^\top \dot{S}(\alpha(t))(x(t) - x(t - \tau(t))) \\
 & - 2(x(t) - x(t - \tau(t)))^\top Q(\alpha(t))x(t - \tau(t)) + (\eta_k - \tau(t))(2\dot{x}^\top(t)Q(\alpha(t))x(t - \tau(t)) \\
 & + 2(x(t) - x(t - \tau(t)))^\top \dot{Q}(\alpha(t))x(t - \tau(t))) + (\eta_k - 2\tau(t))x^\top(t - \tau(t))Ux(t - \tau(t)) \\
 & + (\eta_k - \tau(t))\dot{x}^\top(t)R\dot{x}(t) - \int_{t-\tau(t)}^t \dot{x}^\top(s)R\dot{x}(s)ds.
 \end{aligned} \tag{5.15}$$

By applying Jensen's inequality (Briat, 2011, Eq. (9)), the last term in (5.15) can be bounded as follows:

$$- \int_{t-\tau(t)}^t \dot{x}^\top(s)R\dot{x}(s)ds \leq -2\xi^\top L(\alpha(t))(x(t) - x(t - \tau(t))) + \tau(t)\xi^\top L(\alpha(t))R^{-1}L^\top(\alpha(t))\xi,$$

where $L(\alpha(t)) \in \mathbb{R}^{3n \times n}$, and $\xi = \begin{bmatrix} x^\top(t) & x^\top(t - \tau(t)) & \dot{x}^\top(t) \end{bmatrix}^\top$. Hence, it can be written:

$$\dot{W}(x(t), \alpha(t), \tau(t), \eta_k) \leq \xi^\top \mathcal{Q}(\tau(t))\xi, \tag{5.16}$$

with $\mathcal{Q}(\tau(t)) = \hat{\Pi}_0 + (\eta_k - \tau(t))\Pi_1 + (\eta_k - 2\tau(t))\Pi_2 + \tau(t)(L(\alpha(t))R^{-1}L^\top(\alpha(t)))$,

$$\begin{aligned}
 \hat{\Pi}_0 = & \text{He} \left\{ E_1^\top P(\alpha(t))E_3 - E_{12}^\top Q(\alpha(t))E_2 - L(\alpha(t))E_{12} \right\} - E_{12}^\top S(\alpha(t))E_{12} + E_1^\top \dot{P}(\alpha(t))E_1, \\
 \Pi_1 = & \text{He} \left\{ E_3^\top (Q(\alpha(t))E_2 + S(\alpha(t))E_{12}) + E_{12}^\top \dot{Q}(\alpha(t))E_2 \right\} + E_3^\top R E_3 + E_{12}^\top \dot{S}(\alpha(t))E_{12}, \\
 & \text{and } \Pi_2 = E_2^\top U E_2.
 \end{aligned}$$

From (5.12), consider $\mathcal{B} = \begin{bmatrix} -A(\alpha(t)) & -B(\alpha(t))(K(\alpha(t)) + \tau(t)\dot{K}(\alpha(t - c(t)))) & \mathbf{I} \end{bmatrix}$, such that $\mathcal{B}\xi(t) = 0$, and let $\mathcal{X} = X^\top(\epsilon E_1^\top + E_3^\top)$, with $X \in \mathbb{R}^{n \times n}$. Using the Finsler's Lemma (de Oliveira and Skelton, 2001), the negativeness of $\mathcal{Q}(\tau(t)) + \text{He}\{\mathcal{X}\mathcal{B}\} < 0$ ensures (5.16). Moreover, block (3, 3) of the last inequality requires $X + X^\top + R < \mathbf{0}$ with $\mathbf{0} < R$, thus assuring the nonsingularity of X . Then, applying the similarity transformation $\tilde{\xi} = \Theta\xi$, with $\Theta = \text{diag}\{X, X, X\}$, the inequality (5.16) can be written as $\dot{W}(x(t), \alpha(t), \tau(t), \eta_k) \leq \tilde{\xi}^\top \tilde{\mathcal{Q}}(\tau(t))\tilde{\xi}$ with

$$\tilde{\mathcal{Q}}(\tau(t)) = \Theta^{-\top} \left[\Pi_0 + (\eta_k - \tau(t))\Pi_1 + (\eta_k - 2\tau(t))\Pi_2 + \tau(t)(L(\alpha(t))R^{-1}L^\top(\alpha(t)) + \Pi_3) \right] \Theta^{-1},$$

where $\Pi_0 = \hat{\Pi}_0 + \text{He}\{(\epsilon E_1^\top + E_3^\top)(-XA(\alpha(t))E_1 - XB(\alpha(t))K(\alpha(t))E_2 + XE_3)\}$ and $\Pi_3 = \text{He}\{\epsilon E_1^\top XB(\alpha(t))\dot{K}(\alpha(t - c(t)))E_2 + E_3^\top XB(\alpha(t))\dot{K}(\alpha(t - c(t)))E_2\}$. With the linearity of $\tilde{\mathcal{Q}}(\tau(t))$ in $\tau(t)$ and replacing $\tilde{X} = X^{-1}$, it is possible to change the variables as follows:

$$\begin{aligned}
 \tilde{P}(\alpha(t)) &= \tilde{X}^\top P(\alpha(t))\tilde{X}, & \tilde{S}(\alpha(t)) &= \tilde{X}^\top S(\alpha(t))\tilde{X}, \\
 \tilde{Q}(\alpha(t)) &= \tilde{X}^\top Q(\alpha(t))\tilde{X}, & \tilde{U} &= \tilde{X}^\top U \tilde{X}, \\
 \tilde{L}(\alpha(t)) &= \Theta^{-\top} L(\alpha(t))\tilde{X}, & \tilde{R} &= \tilde{X}^\top R \tilde{X}, \\
 \tilde{K}(\alpha(t)) &= K(\alpha(t))\tilde{X},
 \end{aligned}$$

which yields (5.13) for $\tau(t) = 0$. Next, inequality (5.14) is obtained by assuming $\tau(t) = \eta_k$ and applying the Schur's complement.

Therefore, the feasibility of (5.13)-(5.14) implies (5.6) and (5.8) for $\eta_k \in [\mathcal{T}_1, \mathcal{T}_2]$, with

$$\kappa_1 = \min_{i=1,\dots,N} \text{real}(\lambda(P_i)) > 0, \quad \kappa_2 = \max_{i=1,\dots,N} \text{real}(\lambda(P_i)) > 0,$$

$\forall i = 1, \dots, N$, with $\lambda(P_i)$ being the eigenvalues of P_i , and some sufficiently small $\kappa_3 > 0$, completing the proof. $\blacksquare$

Remark 5.2. *It is important to note that $\tilde{\Pi}_3$ is the only term related to the mismatch between the time-varying parameters of the plant and the controller, as seen in (5.14). This is a key result of this chapter, as other methods typically account for the mismatch in both inequalities (5.13) and (5.14) through a polytopic structure. In addition, by leveraging the mean value theorem, the approach ensures that $\tilde{\Pi}_3$ vanishes when $\tau(t) = 0$, meaning that no mismatch occurs when the sampled-data controller updates the parameters. In contrast, approaches such as those in Gomes da Silva Jr. et al. (2015); Hooshmandi et al. (2018a,b, 2020); Jung et al. (2021) distribute the mismatch across all parameter-dependent matrices (system, looped-functional, etc.), introducing additional conservatism.*

According to Chesi et al. (2004), the space in which $\dot{\alpha}_i(t)$, $i = 1, \dots, N$, varies is bounded by the intersection of the linear constraint $\sum_{i=1}^N \dot{\alpha}_i(t) = 0$ and known bounds on the rate of change. This results in a convex polytopic region defined by

$$\mathcal{D} = \left\{ \phi \in \mathbb{R}^N : \phi = \sum_{\ell=1}^M \beta_{\ell} h_{\ell}^{\ell}, \sum_{i=1}^N h_i^{\ell} = 0, \beta \in \Gamma_M \right\}, \ell = 1, \dots, M,$$

where the vectors h^{ℓ} are the vertices of the polytope that represent the parameter variation rates. This representation considers the exact domain of variation $\alpha(t)$ and $\dot{\alpha}(t)$. Hence, the following expression holds:

$$\dot{Y}(\alpha(t)) = \sum_{i=1}^N \sum_{\ell=1}^M \beta_{\ell} h_i^{\ell} Y_i, \beta \in \Gamma_M, Y \in \{\tilde{K}, \tilde{P}, \tilde{S}, \tilde{Q}\}.$$

The conditions in Theorem 5.2 depend polynomially on the parameters and can be solved numerically using a finite set of LMIs within a convex structure. A parameter-dependent LMI $\Xi(\alpha(t), \dot{\alpha}(t), \dot{\alpha}(t - c(t))) < 0$ can thus be rewritten as:

$$\sum_{i=1}^N \alpha_i(t) \Xi_i + \sum_{j=1}^N \sum_{\ell=1}^M \sum_{m=1}^M \beta_{\ell} h_j^{\ell} \gamma_m g_{m(j)} \Xi_j < \mathbf{0},$$

for all $(\alpha(t), \dot{\alpha}(t), \dot{\alpha}(t - c(t))) \in \Gamma_N \times \Gamma_M \times \Gamma_M$. Note that $\dot{\alpha}(t)$ and $\dot{\alpha}(t - c_i(t))$ share the same polytopic vertices but exist in different simplexes.

In this chapter, the parser ROLMIP is used to automatically convert the parameter-dependent conditions into a finite set of LMIs. Such a conversion is performed by the homogenization of inequalities entries in terms of $\alpha(t)$, allowing an inequality

$$\Upsilon(\alpha(t)) \equiv \sum_{i=1}^N \sum_{j=1}^N \alpha_i \alpha_j \Upsilon_{ij} < \mathbf{0}$$

to be rewritten as

$$\sum_{i=1}^N \alpha_i^2 \Upsilon_{ii} + \sum_{i=1}^{N-1} \sum_{j=i+1}^N \alpha_i \alpha_j (\Upsilon_{ij} + \Upsilon_{ji}) < \mathbf{0},$$

where the coefficients of all monomials are imposed negative definite (Agulhari et al., 2019). Additionally, homogeneous parameter-dependent functions and Pólya's relaxations can be jointly applied, as discussed in the following remark.

Remark 5.3. *The approach enables the derivation of LMI conditions in two ways. The first method involves employing homogeneous parameter-dependent looped-functional candidates for a given degree $d \geq 1$. This allows for a more general controller gain that can depend rationally on the parameters $\alpha(t)$, as proposed by Figueiredo et al. (2021), particularly when $\tilde{X}$ in Theorem 5.2 is a function of $\alpha(t)$. The second method utilizes the Pólya's approach, as described in Oliveira and Peres (2005), which retains the original controller structure while enhancing the estimates of $\mathcal{T}_2$ by introducing new LMIs. This is achieved through the multiplication of the parameter-dependent conditions by $(\alpha_1(t) + \dots + \alpha_N(t))^g$, where $g \geq 0$ represents the Pólya relaxation degree. Both approaches can be readily implemented using the parser ROLMIP (Agulhari et al., 2019).*

5.1.3 Illustrative Example

Consider the LPV system borrowed from Gomes da Silva Jr. et al. (2015), which is commonly used in sampled-data control studies for comparative analysis. The system is given by:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0.1 & 0.4 + 0.6\rho(t) \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad (5.17)$$

where $|\rho(t)| \leq 1$. The results from Theorem 5.2 are compared with recent literature to illustrate the improvements of the proposed method. The primary objective is to determine the maximum sampling value, $\mathcal{T}_2$, with $\mathcal{T}_1 = 0.001$ s, for various bounds on the parameter variation rate $|\dot{\rho}(t)| \leq \sigma$, i.e., for $\sigma \in \{0.2, 0.6, 1\}$.

To express the matrices that depend affinely on the bounded parameter $\rho(t)$ in a polytopic representation, it follows that $\dot{x}(t) = \mathcal{A}(\rho(t))x(t)$ with $\mathcal{A}(\rho(t)) = \mathcal{A}_0 + \rho(t)\mathcal{A}_1$. This leads to the polytopic form $\dot{x}(t) = A(\alpha(t))x(t)$, where $A(\alpha(t)) = \alpha_1(t)A_1 + \alpha_2(t)A_2$. The vertices are defined as $A_1 = \mathcal{A}_0 - \mathcal{A}_1$ and $A_2 = \mathcal{A}_0 + \mathcal{A}_1$, with $\alpha_1(t) = (1 - \rho(t))/2$ and $\alpha_2(t) = (1 + \rho(t))/2$.

The bounds on the variation rates of the time-varying parameter $\rho(t)$ translate to bounds on the variation rates of the simplex representation as $|\dot{\alpha}_i(t)| = |(-1)^i \dot{\rho}(t)/2| \leq \sigma/2$ for $i = 1, 2$, subject to $\dot{\alpha}_1(t) + \dot{\alpha}_2(t) = 0$. As previously mentioned, the polytopic representation, including $\alpha_i(t)$ and $\dot{\alpha}_i(t)$, can be automatically obtained using the parser ROLMIP (Agulhari et al., 2019).

The results are compared with those in Gomes da Silva Jr. et al. (2015); Hooshmandi et al. (2018a,b, 2020); Jung et al. (2021); Wei et al. (2022). Notably, these works utilize the polytopic modeling approach proposed by Gomes da Silva Jr. et al. (2015), which assigns the intersampling time-varying parameter to a second polytope. As a result, all matrices that depend on this parameter (including system and looped-functional ones) are affected, requiring

the combination of the polytopes corresponding to $\alpha(t)$, $\alpha(t_k)$, and $\dot{\alpha}(t)$ for LMI feasibility. In contrast, Theorem 5.2 avoids this complexity, as discussed in Remark 5.2.

To clarify the comparison, it is noted that Gomes da Silva Jr. et al. (2015) employs a looped-functional similar to the one used in this section, but with constant matrices in $\mathcal{V}_0$. In contrast, Jung et al. (2021) and Hooshmandi et al. (2020) utilize a looped-functional that is polynomial in the parameters, leading to sum-of-squares (SOS) conditions dependent on a polynomial of degree $2d$ and demanding more computational effort to be solved than the proposed LMIs as discussed in the sequel. Moreover, Hooshmandi et al. (2018a) and Jung et al. (2021) enhance their results by applying Wirtinger’s inequality instead of Jensen’s inequality. Although yielding less conservative conditions, Wirtinger’s inequality requires more decision variables, leading to greater numerical complexity than Jensen’s inequality.

A grid search over a scalar ($\epsilon > 0$) is conducted to determine $\mathcal{T}_2$. The conditions are solved using Mosek (Andersen and Andersen, 2000). The results are summarized in Table 5.1, where Theorem 5.2 achieves the highest values of $\mathcal{T}_2$ for the examined bounds on $\dot{\rho}(t)$. Moreover, note that in lines 5, 7, and 9, all looped-functional matrices are kept constant for these specific studies. This ensures a fair comparison between the results from Gomes da Silva Jr. et al. (2015), Jung et al. (2021), and Theorem 5.2. Notably, the SOS approach from Jung et al. (2021) yields a robust control gain when $d = 0$. Across all investigated derivative bounds, Theorem 5.2 consistently outperforms the other methods. The method requires 58 decision variables with a parameter-dependent looped-functional, or 36 when the matrices are constant. In comparison, Hooshmandi et al. (2018a), Hooshmandi et al. (2018b), and Wei et al. (2022) require 164, 116, and 177 variables, respectively, demanding greater computational effort. While Gomes da Silva Jr. et al. (2015) also uses 36 variables with constant matrices, it provides smaller sampling interval estimates than the proposed theorem. Methods solving SOS problems are even more demanding, such as Jung et al. (2021), which requires 638 variables for the simplest case with $d = 0$ (Corollary 1).

Table 5.1: Maximum sampling interval, $\mathcal{T}_2$, for different bounds on $\dot{\rho}(t)$ (or equivalently of $\dot{\alpha}_i(t)$) and $\mathcal{T}_1 = 0.001$. Lines 5, 7, and 9 refer to results with parameter-independent looped-functional matrices.

#	Bounds of $\dot{\rho}(t)$ (or $\dot{\alpha}_i(t)$)	0.2 (0.1)	0.6 (0.3)	1 (0.5)
1	(Hooshmandi et al., 2018a, Th. 2)	0.77s	0.73s	0.69s
2	(Hooshmandi et al., 2020, Prop. 3.6)	0.81s	0.80s	0.78s
3	(Hooshmandi et al., 2018b, Th. 2)	0.84s	0.81s	0.77s
4	(Wei et al., 2022, Cor. 1)	0.95s	0.93s	0.92s
5	(Gomes da Silva Jr. et al., 2015, Th. 1), P	1.21s	1.08s	0.76s
6	(Gomes da Silva Jr. et al., 2015, Th. 1), $P(\alpha(t))$	1.26s	1.09s	0.80s
7	(Jung et al., 2021, Cor. 1, $d = 0$)	1.00s	1.00s	1.00s
8	(Jung et al., 2021, Th. 1, $d = 1$)	1.36s	1.23s	1.12s
9	Theorem 5.2 , P, S, L, Q	1.37s	1.19s	1.12s
10	Theorem 5.2 , $P(\alpha(t)), S(\alpha(t)), L(\alpha(t)), Q(\alpha(t))$	1.45s	1.26s	1.18s

Additionally, as mentioned in Remark 5.3, the conditions of Theorem 5.2 can be relaxed using homogeneous parameter-dependent loop functional candidates and Pólya's relaxation. This allows comparison with (Jung et al., 2021, Theorem 1) for $d = 3$, which yields maximum sampling intervals $\mathcal{T}_2$ of 1.41 s, 1.35 s, and 1.29 s for the respective limits of $\dot{\rho}(t)$ shown in Table 5.1. In contrast, the proposed method achieves 1.48 s, 1.36 s, and 1.31 s for $d = 3$ and $g = 3$ (Remark 5.3), demonstrating significant improvements, particularly for slowly varying parameters.

Figure 5.1 illustrates the closed-loop behavior for the initial condition $x(0) = \begin{bmatrix} 2 & 1 \end{bmatrix}^\top$, with $\rho(t) = -\cos(0.2t)$, $|\dot{\rho}(t)| \leq 0.2$, $\mathcal{T}_1 = 0.001$, and $\mathcal{T}_2 = 1.45$. The control gains obtained via Theorem 5.2 are $K_1 = \begin{bmatrix} -0.1228 & -0.5763 \end{bmatrix}$ and $K_2 = \begin{bmatrix} -0.1076 & -1.2343 \end{bmatrix}$. The top plot shows the states of the system, while the bottom plot displays the control signal (scaled on the right) alongside the aperiodic sampling intervals η_k (scaled on the left), limited by the dashed line representing $\mathcal{T}_2$. The zoomed-in detail of the first 20 seconds clearly illustrates the control signal updates. This simulation demonstrates how the designed sampled-data controller stabilizes the system despite parameter mismatches and random sampling intervals within the specified bounds. In contrast, the other compared methods fail to achieve a viable controller for this scenario. Overall, this example highlights the superior estimates achieved by Theorem 5.2

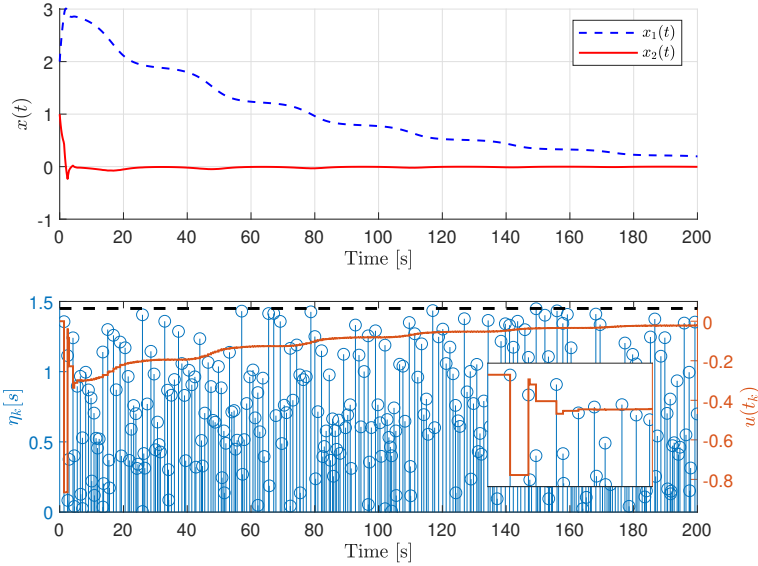


Figure 5.1: Top plot: states of the system, $x(t)$. Bottom plot: random sampling intervals $\eta_k \in [0.001, 1.450]$ s and the sampled-data control signal $u(t_k)$.

compared to recent literature, even when those works employ more complex methods.

5.2 Robust Stabilization of LPV Sampled-Data Systems via Quadratic Polynomial Conditions

As discussed in Chapter 4, the choice of an appropriate looped functional and its time derivative is crucial for reducing the conservatism of solutions. First, consider the robust controller case. The motivation for employing a robust controller rather than a parameter-dependent

one stems from the unrealistic assumptions regarding parameter availability. In [Peixoto et al. \(2024\)](#), co-design conditions for parameter-dependent controllers and event-triggered mechanisms in LPV systems were proposed to address denial-of-service and stochastic deception attacks. Such an approach assumes that the time-varying parameters of the controller evolve continuously, whereas only the system states are sampled and transmitted through event-triggered mechanisms. However, this assumption is often unrealistic in practice, since both states and parameters must typically be sampled in real-world applications. In contrast, the robust control strategy adopted in this section avoids such restrictive assumptions and also offers a simpler implementation.

Consider the continuous-time LPV system given in (5.1) and a control law defined as:

$$u(t) = Kx(t_k), \quad (5.18)$$

with $t \in [t_k, t_{k+1})$, and $K \in \mathbb{R}^{n_u \times n}$ is the robust gain. The resulting closed-loop system is represented by:

$$\dot{x}(t) = A(\alpha(t))x(t) + B(\alpha(t))Kx(t_k). \quad (5.19)$$

Thus, the problem to be investigated is defined as follows:

Problem 5.2. *Given the continuous-time LPV system represented by (5.1) and the sampled-state feedback control law (5.18), design the robust gain K such that the maximum allowable sampling interval is maximized, and the closed-loop system is asymptotically stable for all $\eta_k \in [\mathcal{T}_1, \mathcal{T}_2]$.*

5.2.1 Second-Order Looped-Functional Conditions

To obtain less conservative conditions, it is assumed that the derivative of the looped-functional results in a quadratic polynomial in $\tau(t)$. The following looped-functional that satisfies (5.6) and (5.7) is proposed:

$$V(x(t), \alpha(t)) = x^\top(t)P(\alpha(t))x(t), \quad (5.20)$$

$$\begin{aligned} \mathcal{V}_0(x(t), \alpha(t), \tau(t), \eta_k) &= (\eta_k - \tau(t)) \left[(x(t) - x(t - \tau(t)))^\top \left(S(\alpha(t))(x(t) - x(t - \tau(t))) \right. \right. \\ &\quad \left. \left. + 2Q(\alpha(t))x(t - \tau(t)) \right) + \tau(t)\zeta^\top U\zeta + \int_{t-\tau(t)}^t \dot{x}^\top(s)R\dot{x}(s)ds \right], \end{aligned} \quad (5.21)$$

where $\zeta = \begin{bmatrix} x^\top(t) & x^\top(t - \tau(t)) \end{bmatrix}^\top$, with $\mathbf{0} < P(\alpha(t)) \in \mathbb{S}^n$, $\mathbf{0} < R \in \mathbb{S}^n$, $S(\alpha(t)) \in \mathbb{S}^n$, $U \in \mathbb{S}^n$, and $Q(\alpha(t)) \in \mathbb{R}^{n \times n}$. The proposed looped-functional $\mathcal{W}(x(t), \alpha(t), \tau(t), \eta_k) = V(x(t), \alpha(t)) + \mathcal{V}_0(x(t), \alpha(t), \tau(t), \eta_k)$ is based on the one originally presented by [Seuret \(2012\)](#), extended with parameter-dependent matrices by [Gomes da Silva Jr. et al. \(2018\)](#) and used in the previous section. The term $\zeta^\top U\zeta$ is modified by introducing $x(t)$ in ζ , resulting in its derivative being a second-order polynomial in $\tau(t)$, as presented in the stability analysis case in Chapter 4.

Remark 5.4. Note that the looped-functional (5.20)–(5.21) can be parameter-independent by setting $P(\alpha(t)) = P$, $S(\alpha(t)) = S$, and $Q(\alpha(t)) = Q$, consequently resulting in the corresponding derivatives being equal to zero. This can lead to a more conservative formulation, based on the quadratic stability approach.

5.2.2 Negative Definiteness Lemmas

To establish the desired conditions using the proposed looped-functional, it is necessary to ensure the negativity of the respective derivative. This is achieved by employing negativity lemmas for quadratic functions, which provide sufficient conditions for negative definiteness. For this reason, negativity lemmas for quadratic functions, explored in Chapter 4, are recalled.

First, to demonstrate the effectiveness of the proposed method, the necessary and sufficient negativity lemma for the precisely known case, based on the S-Procedure proposed by [Park and Park \(2020\)](#) (presented in Lemma 5.2), is considered for comparison purposes:

Lemma 5.2 ([Park and Park \(2020\)](#)). Let $f(y) = \mathcal{A}_2 y^2 + \mathcal{A}_1 y + \mathcal{A}_0$ be a quadratic function, where $\mathcal{A}_0, \mathcal{A}_1, \mathcal{A}_2 \in \mathbb{R}^{p \times p}$. If there exists a matrix $M \in \mathbb{R}^{p \times p}$ with $M + M^\top \geq \mathbf{0}$ such that

$$\begin{bmatrix} f(\underline{h}) & \frac{1}{2}\mathcal{A}_1 + \underline{h}\mathcal{A}_2 + (\bar{h} - \underline{h})M \\ \star & \mathcal{A}_2 - M - M^\top \end{bmatrix} < \mathbf{0}, \quad (5.22)$$

then $f(y) < 0, \forall y \in [\underline{h}, \bar{h}]$.

Next, consider the use of Pólya's Theorem-based condition (presented in 5.3) with $N_p = 1$, to satisfy the quadratic negativity:

Lemma 5.3. For all $\theta \in \Gamma_2$ and let $f(y) = \mathcal{A}_2 y^2 + \mathcal{A}_1 y + \mathcal{A}_0$ be a quadratic function, where $\mathcal{A}_0, \mathcal{A}_1, \mathcal{A}_2 \in \mathbb{R}^{p \times p}$. For a sufficiently large $d \in \mathbb{N}$, $f(y) < 0 \forall y \in [\underline{h}, \bar{h}]$ if all the coefficients of the monomials in

$$\left(\theta_1^2 f(\bar{h}) + \theta_1 \theta_2 \mathcal{F}(\underline{h}, \bar{h}) + \theta_2^2 f(\underline{h}) \right) (\theta_1 + \theta_2)^d, \quad (5.23)$$

with $\mathcal{F}(\underline{h}, \bar{h}) = (2\underline{h}\bar{h}\mathcal{A}_2 + (\underline{h} + \bar{h})\mathcal{A}_1 + 2\mathcal{A}_0)$, are strictly negative.

Lemmas 5.2 and 5.3 are necessary and sufficient for cases where the coefficients of the polynomial $\mathcal{A}_2, \mathcal{A}_1$, and $\mathcal{A}_0$ are independent of parameters. However, this chapter aims to investigate the use of such lemmas in LPV systems, i.e., with parametric dependence on the coefficients. Lemma 5.2 presents fewer inequalities to be verified than Lemma 5.3, although it includes additional decision variables. Furthermore, Lemma 5.3 allows the relaxation of Pólya by increasing the degree d , at the cost of increasing the number of LMIs, as discussed in Chapter 4.

5.2.3 Robust Stabilization Conditions

The following theorem provides a solution to Problem 5.2.

Theorem 5.3. If there exists matrices $\mathbf{0} < \tilde{P}(\alpha(t)) \in \mathbb{S}^n$, $\mathbf{0} < \tilde{R} \in \mathbb{S}^n$, $\tilde{S}(\alpha(t)) \in \mathbb{S}^n$, $\tilde{Q}(\alpha(t)) \in \mathbb{R}^{n \times n}$, $\tilde{U} \in \mathbb{S}^{2n}$, $\tilde{L}(\alpha(t)) \in \mathbb{R}^{3n \times n}$, $\tilde{K} \in \mathbb{R}^{n_u \times n}$, $\tilde{X} \in \mathbb{R}^{n \times n}$, and a scalar $\epsilon > 0$ satisfying

$$\tau^2(t)\tilde{\Pi}_2 + \tau(t)\tilde{\Pi}_1(\alpha(t)) + \tilde{\Pi}_0(\alpha(t)) < \mathbf{0}, \forall \tau(t) \in [0, \mathcal{T}_i], \quad (5.24)$$

$\forall i = 1, 2$, with

$$\begin{aligned}\tilde{\Pi}_2 &= -\text{He}\{\bar{E}_1^\top \tilde{U} \bar{E}_2\}, \\ \tilde{\Pi}_1(\alpha(t)) &= \text{He}\left\{-E_3^\top \left(\tilde{Q}(\alpha(t))E_2 + \tilde{S}(\alpha(t))E_{12}\right) - E_{12}^\top \dot{\tilde{Q}}(\alpha(t))E_2\right\} - \mathcal{T}_i \tilde{\Pi}_2 - 2\bar{E}_1^\top \tilde{U} \bar{E}_1 - E_3^\top \tilde{R}E_3 \\ &\quad - E_{12}^\top \dot{\tilde{S}}(\alpha(t))E_{12} + \tilde{L}(\alpha(t))\tilde{R}^{-1}\tilde{L}^\top(\alpha(t)), \\ \tilde{\Pi}_0(\alpha(t)) &= \text{He}\left\{E_1^\top \tilde{P}(\alpha(t))E_3 - E_{12}^\top \tilde{Q}(\alpha(t))E_2 - \tilde{L}(\alpha(t))E_{12} + (\epsilon E_1^\top + E_3^\top)(-A(\alpha(t))\tilde{X}E_1 - \right. \\ &\quad \left. B(\alpha(t))\tilde{K}E_2 + \tilde{X}E_3) + \mathcal{T}_i E_3^\top (\tilde{Q}(\alpha(t))E_2 + \tilde{S}(\alpha(t))E_{12}) + \mathcal{T}_i E_{12}^\top \dot{\tilde{Q}}(\alpha(t))E_2\right\} \\ &\quad + \mathcal{T}_i (E_3^\top \tilde{R}E_3 + \bar{E}_1^\top \tilde{U} \bar{E}_1 + E_{12}^\top \dot{\tilde{S}}(\alpha(t))E_{12}) - E_{12}^\top \tilde{S}(\alpha(t))E_{12} + E_1^\top \dot{\tilde{P}}(\alpha(t))E_1,\end{aligned}$$

where $E_1 = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix}$, $E_2 = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \end{bmatrix}$, $E_3 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}$, $E_{12} = E_1 - E_2$, $\bar{E}_1 = \begin{bmatrix} E_1 & E_2 \end{bmatrix}^\top$, $\bar{E}_2 = \begin{bmatrix} E_3 & \mathbf{0} \end{bmatrix}^\top$. Then, system (5.1) is asymptotically stable with the sampled-data controller (5.18), whose robust gain matrix is given by

$$K = \tilde{K} \tilde{X}^{-1},$$

for any sampling sequences satisfying $0 < \mathcal{T}_1 \leq \eta_k \leq \mathcal{T}_2$.

Proof: Consider a parameter-dependent looped-functional given by (5.20) and (5.21), the corresponding time derivative results in:

$$\begin{aligned}\dot{\mathcal{W}}(x(t), \alpha(t), \tau(t), \eta_k) &= 2\dot{x}^\top(t)P(\alpha(t))x(t) + x^\top(t)\dot{P}(\alpha(t))x(t) \\ &\quad - (x(t) - x(t - \tau(t)))^\top S(\alpha(t))(x(t) - x(t - \tau(t))) \\ &\quad + (\eta_k - \tau(t))2\dot{x}^\top(t)S(\alpha(t))(x(t) - x(t - \tau(t))) \\ &\quad + (\eta_k - \tau(t))(x(t) - x(t - \tau(t)))^\top \dot{S}(\alpha(t))(x(t) - x(t - \tau(t))) \\ &\quad - 2(x(t) - x(t - \tau(t)))^\top Q(\alpha(t))x(t - \tau(t)) + (\eta_k - \tau(t))(2\dot{x}^\top(t)Q(\alpha(t))x(t - \tau(t)) \\ &\quad + 2(x(t) - x(t - \tau(t)))^\top \dot{Q}(\alpha(t))x(t - \tau(t))) - \tau(t)\zeta^\top U\zeta + (\eta_k - \tau(t))\zeta^\top U\zeta \\ &\quad + (\eta_k - \tau(t))\tau(t)2\dot{\zeta}^\top U\zeta + (\eta_k - \tau(t))\dot{x}^\top(t)R\dot{x}(t) - \int_{t-\tau(t)}^t \dot{x}^\top(s)R\dot{x}(s)ds,\end{aligned}$$

where $\dot{\zeta} = \begin{bmatrix} \dot{x}^\top(t) & \mathbf{0} \end{bmatrix}^\top$. Note that the last term of $\dot{\mathcal{W}}(x(t), \alpha(t), \tau(t), \eta_k)$ can be upper bounded using the inequality (Briat, 2011, Eq. (9)) as

$$- \int_{t-\tau(t)}^t \dot{x}^\top(s)R\dot{x}(s)ds \leq -2\xi^\top L(\alpha(t))(x(t) - x(t - \tau(t))) + \tau(t)\xi^\top L(\alpha(t))R^{-1}L^\top(\alpha(t))\xi,$$

where $\mathbf{0} < R \in \mathbb{S}^n$, $L(\alpha) \in \mathbb{R}^{3n \times n}$ and $\xi = \begin{bmatrix} x^\top(t) & x^\top(t - \tau(t)) & \dot{x}^\top(t) \end{bmatrix}^\top$. Thus, it follows that

$$\dot{\mathcal{W}}(x(t), \alpha(t), \tau(t), \eta_k) \leq \xi^\top \mathcal{Q}(\tau(t))\xi, \quad (5.25)$$

with $\mathcal{Q}(\tau(t)) = \tau^2(t)\Pi_2 + \tau(t)\Pi_1(\alpha(t)) + \hat{\Pi}_0(\alpha(t))$,

$$\begin{aligned}\Pi_2 &= -\text{He}\left\{\bar{E}_1^\top U \bar{E}_2\right\}, \\ \Pi_1(\alpha(t)) &= \text{He}\left\{-E_3^\top (Q(\alpha(t))E_2 + S(\alpha(t))E_{12}) - E_{12}^\top \dot{Q}(\alpha(t))E_2\right\} - \eta_k \Pi_2 - 2\bar{E}_1^\top U \bar{E}_1 - E_3^\top R E_3 \\ &\quad - E_{12}^\top \dot{S}(\alpha(t))E_{12} + L(\alpha)R^{-1}L(\alpha(t))^\top, \\ \hat{\Pi}_0(\alpha(t)) &= \text{He}\left\{E_1^\top P(\alpha(t))E_3 - E_{12}^\top Q(\alpha(t))E_2 - L(\alpha(t))E_{12} + \eta_k E_3^\top (Q(\alpha(t))E_2 + S(\alpha(t))E_{12})\right. \\ &\quad \left.+ \eta_k E_{12}^\top \dot{Q}(\alpha(t))E_2\right\} + \eta_k (E_3^\top R E_3 + \bar{E}_1^\top U \bar{E}_1 + E_{12}^\top \dot{S}(\alpha(t))E_{12}) - E_{12}^\top S(\alpha(t))E_{12} + E_1^\top \dot{P}(\alpha(t))E_1,\end{aligned}$$

From (5.19), it follows that $\mathcal{B}\xi(t) = 0$ where $\mathcal{B} = \begin{bmatrix} -A(\alpha(t)) & -B(\alpha(t))K & \mathbf{I} \end{bmatrix}$ and $\mathcal{X} = \begin{bmatrix} \epsilon X & \mathbf{0} & X \end{bmatrix}^\top$, with $X \in \mathbb{R}^{n \times n}$. Using Finsler's Lemma (de Oliveira and Skelton, 2001), and a similarity transformation $\tilde{\xi} = \Theta\xi$, where $\Theta = \text{diag}\{X, X, X\}$, equation (5.25) can be rewritten as $\dot{\mathcal{W}}(x(t), \alpha(t), \tau(t), \eta_k) \leq \tilde{\xi}^\top \tilde{\mathcal{Q}}(\tau(t))\tilde{\xi}$, with

$$\tilde{\mathcal{Q}}(\tau(t)) = \Theta^{-\top} \left(\tau^2(t)\Pi_2 + \tau(t)\Pi_1(\alpha(t)) + \Pi_0(\alpha(t)) \right) \Theta^{-1},$$

where $\Pi_0(\alpha(t)) = \hat{\Pi}_0(\alpha(t)) + \text{He}\{\mathcal{X}\mathcal{B}\}$. Hence, assuming $\tilde{X} = X^{-1}$, the following change of variables is applied:

$$\begin{aligned}\tilde{P}(\alpha(t)) &= \tilde{X}^\top P(\alpha(t))\tilde{X}, & \tilde{S}(\alpha(t)) &= \tilde{X}^\top S(\alpha(t))\tilde{X}, \\ \tilde{Q}(\alpha(t)) &= \tilde{X}^\top Q(\alpha(t))\tilde{X}, & \tilde{U} &= \text{diag}\{\tilde{X}, \tilde{X}\}^\top U \tilde{X}, \\ \tilde{L}(\alpha(t)) &= \Theta^{-\top} L(\alpha(t))\tilde{X}, & \tilde{R} &= \tilde{X}^\top R \tilde{X}, \\ \tilde{K} &= K \tilde{X}.\end{aligned}$$

Therefore, the feasibility of (5.24) implies (5.6) and (5.8), for $\eta_k \in [\mathcal{T}_1, \mathcal{T}_2]$, with

$$\kappa_1 = \min_{i=1,\dots,N} \text{real}(\lambda(P_i)) > 0, \quad \kappa_2 = \max_{i=1,\dots,N} \text{real}(\lambda(P_i)) > 0,$$

$\forall i = 1, \dots, N$, with $\lambda(P_i)$ being the eigenvalues of P_i , and some sufficiently small $\kappa_3 > 0$. It is noted that the (3,3) block of $\tilde{\Pi}_0$ satisfies $\tilde{X} + \tilde{X}^\top + \tilde{R} < \mathbf{0}$. Since $\tilde{R} > \mathbf{0}$, this implies that $\tilde{X} + \tilde{X}^\top < \mathbf{0}$. Therefore, $\tilde{X}$ is nonsingular, completing the proof. ■

To ensure the conditions of Theorem 5.3, a negative definiteness lemma must be employed, as Lemmas 5.2 and 5.3, and the Schur complement is required to assess $\tilde{\Pi}_1(\alpha(t))$ in a convex formulation due to $\tilde{L}(\alpha(t))\tilde{R}^{-1}\tilde{L}^\top(\alpha(t))$.

5.2.4 Illustrative Examples

The numerical examples are implemented using the ROLMIP parser (Aguilhari et al., 2019) and the Mosek solver (Andersen and Andersen, 2000). The LMIs from Theorem 5.3 with Lemmas 5.2 and 5.3 are solved iteratively by testing feasibility for increasing values of $\mathcal{T}_2$ until the maximum feasible value is obtained through a linear search on the scalar parameter $\epsilon \in [0, 1]$, with a step size of 0.01.

Example 1

Consider the LPV system borrowed from [Ramezanifar et al. \(2012\)](#):

$$\dot{x}(t) = \begin{bmatrix} 2\rho(t) & 1.1 + \rho(t) \\ -2.2 + \rho(t) & -3.3 + 0.1\rho(t) \end{bmatrix} x(t) + \begin{bmatrix} 2\rho(t) \\ 0.1 + \rho(t) \end{bmatrix} u(t),$$

with $\rho(t) \in [-1, 1]$ and $\dot{\rho}(t) \in [-0.2, 0.2]$. Assuming a minimum sampling time of $\mathcal{T}_1 = 0.001$ s, a linear search over ϵ is performed to obtain the maximum feasible $\mathcal{T}_2$. Considering particular cases in which the functional is parameter-independent, as proposed in Remark 5.4, Theorem 5.3 jointly with Lemma 5.3 with $d = 0$ and $\epsilon = 0.87$ yields a maximum sampling time of $\mathcal{T}_2 = 0.851$ s, with 40 scalar decision variables. On the other hand, Theorem 5.3 jointly with Lemma 5.2 with $\epsilon = 0.82$ results in $\mathcal{T}_2 = 0.768$ s with 112 scalar variables (10% lower in $\mathcal{T}_2$ and 280% more variables). This example highlights the superiority of Lemma 5.3 with respect to Lemma 5.2, achieving less conservative results even with fewer decision variables.

Example 2

Consider the LPV system borrowed from [Gomes da Silva Jr. et al. \(2015\)](#), in which

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0.1 & 0.4 + 0.6\rho(t) \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad (5.26)$$

with $\rho(t) \in [-1, 1]$ and $\dot{\rho}(t) \in [-b, b]$. Table 5.2 presents the maximum values of $\mathcal{T}_2$, in seconds, obtained with $\mathcal{T}_1 = 0.001$ s, for different bounds on $\dot{\rho}(t)$, using Theorem 5.3 with Lemmas 5.3 and 5.2, in comparison with some results from the literature.

Table 5.2: Maximum value of $\mathcal{T}_2$ (Ex. 2).

Condition	$b = 0.2$	$b = 0.6$	$b = 1$
(Hooshmandi et al., 2020)	0.806	0.795	0.781
(Hooshmandi et al., 2018b)	0.838	0.812	0.773
(Gomes da Silva Jr. et al., 2015)	1.264	1.002	0.800
Theorem 5.3 with Lemma 5.2	1.382	1.227	1.157
Theorem 5.3 with Lemma 5.3, $d = 0$	1.383	1.233	1.166
Theorem 5.3 with Lemma 5.3, $d = 1$	1.384	1.233	1.167
Theorem 5.3 with Lemma 5.3, $d = 5$	1.384	1.234	1.168

Note that even using a robust gain, Theorem 5.3 yields a larger allowable sampling time compared to the referenced works that employ parameter-dependent controllers. Likewise, the proposed results are the only ones among those listed that use a functional whose derivative results in a second-order polynomial. Therefore, it is observed that using such functionals can lead to less conservative synthesis conditions. Furthermore, the higher the degree of Pólya's relaxation, d , the greater the maximum admissible sampling time achieved using Theorem 5.3 with Lemma 5.3, which requires 62 scalar decision variables. Theorem 5.3 with Lemma 5.2 requires 134 scalar decision variables, highlighting the superiority of Lemma 5.3, which provides less conservative results with lower computational cost.

Example 3

Consider the model of a helicopter in vertical flight subject to changes in airspeed, represented by time-varying parameters, borrowed from (Narendra and Tripathi, 1973). The state variables are the horizontal velocity ($x_1(t)$), vertical velocity ($x_2(t)$), pitch rate ($x_3(t)$), and pitch angle ($x_4(t)$). The control inputs are the collective pitch control ($u_1(t)$) and longitudinal cyclic pitch control ($u_2(t)$). The system matrices are given by:

$$A(\rho_1(t), \rho_2(t)) = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.0100 & 0.0024 & -4.0208 \\ 0.1002 & \rho_1(t) & -0.7070 & \rho_2(t) \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B(\rho_3(t)) = \begin{bmatrix} 0.4422 & 0.1761 \\ \rho_3(t) & -7.5922 \\ -5.5200 & 4.9900 \\ 0 & 0 \end{bmatrix},$$

with $\rho_1(t) \in [-0.0664, 0.5047]$, $\rho_2(t) \in [0.1198, 2.526]$, and $\rho_3(t) \in [0.9775, 5.112]$. The respective rates of variation satisfy $\dot{\rho}_1(t) \in [-0.15, 0.15]$, $\dot{\rho}_2(t) \in [-0.8, 0.8]$ and $\dot{\rho}_3(t) \in [-1.7, 1.7]$. Since the system has three time-varying parameters, a polytope with $N = 8$ is obtained in Γ_8 .

Using Theorem 5.3 with Lemma 5.3 with $d = 5$ and considering a minimum sampling time of $\mathcal{T}_1 = 0.1$ s, the system is stable for sampling times up to $\mathcal{T}_2 = 0.864$ s. For simulation purposes, a controller (5.18) is designed via Theorem 5.3 with Lemma 5.3 with $d = 5$ and $\epsilon = 1.2$, which ensures the stability of the closed-loop system (5.19) for sampling instants $\eta_k \in [0.200, 0.518]$ s. The resulting robust gain matrix is:

$$K = \begin{bmatrix} -0.0926 & 0.0382 & 0.4354 & 0.3619 \\ -0.0509 & 0.0250 & 0.1062 & -0.3433 \end{bmatrix}.$$

Considering the time-varying parameter functions as $\rho_1(t) = 0.066 + |0.42\sin 0.35t|$, $\rho_2(t) = 0.11 + |2.25\sin 0.35t|$, and $\rho_3(t) = 0.97 + |4.1\sin 0.35t|$, and initial conditions $x(0) = [2 \ 1 \ -2 \ 1.5]^\top$, the state-feedback is implemented with sampling times η_k randomly generated within the specified interval, $[0.200, 0.518]$ s. Figure 5.2 shows the system states $x(t)$ and the sampling intervals $\eta_k = t_{k+1} - t_k$ generated during the simulation. As stated in Theorem 5.3, the system states converge asymptotically to the origin with sampling times within the specified bounds, illustrating the stabilization of the system under aperiodic sampling.

Figure 5.3 displays the corresponding time-varying parameters within the simplex Γ_8 throughout the simulation, as well as the control signals $u(t)$ from the implemented robust controller. This illustrates the applicability of the approach in simulation on a practical example, considering a helicopter in vertical flight with time-varying parameters. This system is four-dimensional and involves a larger number of vertices, making it a more challenging case and highlighting the capability of the method to handle realistic, higher-order models.

5.2.5 Quasi-LPV Real-Time Experimental Validation

As proposed by Palmeira et al. (2021), additional positivity conditions on the looped functional must be imposed to ensure local stability of *quasi*-LPV systems in the context of sampled-data systems. These extra requirements avoid allowing the parameter to evolve out of its domain

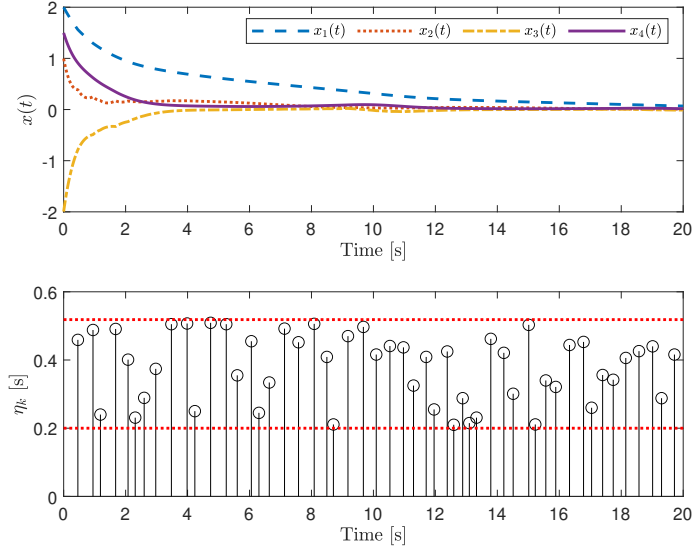


Figure 5.2: Top plot: states of the system, $x(t)$, asymptotically converging to the origin. Bottom plot: random sampling intervals $\eta_k \in [0.200, 0.528]$ s, dashed red lines are the bounds of the intervals.

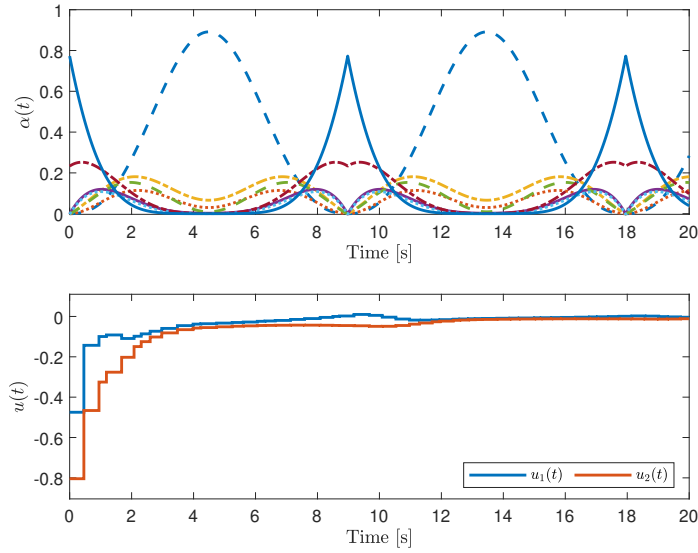


Figure 5.3: Top plot: time-varying parameters $\alpha(t) \in \Gamma_8$. Bottom plot: control signals during the simulation.

during the sampling interval. Then, (5.7) must satisfy

$$\mathcal{V}_0(x, \alpha, 0, \eta_k) = \mathcal{V}_0(x, \alpha, \eta_k, \eta_k) = 0, \quad (5.27)$$

$$\mathcal{V}_0(x, \alpha, \tau, \eta_k) > 0, \forall \tau \in [0, \eta_k], \quad (5.28)$$

where $x(t)$ belong to a compact set $\mathcal{R} \subseteq \mathbb{R}^n$. Therefore, assuming (5.21), condition (5.28) can be satisfied with Theorem 5.3 verifying:

$$\begin{bmatrix} \tilde{S}(\alpha(t)) & \tilde{Q}^\top(\alpha(t)) - \tilde{S}(\alpha(t)) \\ \star & \tilde{S}(\alpha(t)) - \tilde{Q}(\alpha(t)) - Q^\top(\alpha(t)) \end{bmatrix} \geq \mathbf{0}, \quad (5.29)$$

and $\tilde{X} \geq \mathbf{0}$. For details, see (Palmeira et al., 2021, Theorem 1 and Lemma 2).

Since in *quasi*-LPV systems the time-varying parameters depend on the plant states, it is reasonable to assume that the stability is only valid in a subset of the state space given by:

$$\mathcal{R}_x = \{x \in \mathbb{R}^n; |H_{(r)}x| \leq \sigma_{(r)}\}, \quad (5.30)$$

with $\sigma_{(r)} > 0$, $H_{(r)} \in \mathbb{R}^{1 \times n}$, and $r = 1, \dots, r_c$, where r_c is the number of constrains. Therefore, the subset $\mathcal{R}_x$ can be included in $\mathcal{R}$ with the following LMI (Silva et al., 2014):

$$\begin{bmatrix} \tilde{P}(\alpha(t)) & \tilde{X}H_{(r)}^\top \\ \star & \sigma_{(r)}^2 \end{bmatrix} > \mathbf{0}, \quad r = 1, \dots, r_c. \quad (5.31)$$

The local stabilization can be optimized by maximizing the estimate of the region of attraction, considering an ellipsoidal set $\mathcal{E}(R_0) = \{x \in \mathbb{R}^n; x^\top R_0 x \leq 1\} \subseteq \mathcal{R}$, ensuring that $R_0 \geq P(\alpha(t))$, with

$$\begin{bmatrix} R_0 & \mathbf{I} \\ \star & -\tilde{P}(\alpha(t)) - \text{He}\{\tilde{X}\} \end{bmatrix} > \mathbf{0}. \quad (5.32)$$

The control law (5.18) is sought to maximize the set $\mathcal{E}(P(\alpha(t))) = \{x \in \mathbb{R}^n, x^\top P(\alpha(t))x \leq 1\}$ through the following procedure:

$$\begin{aligned} \min \quad & \gamma \\ \text{s.t.} \quad & \text{Theorem 5.3, (5.29), (5.31), and (5.32) with } R_0 = \gamma \mathbf{I}. \end{aligned} \quad (5.33)$$

Nonlinear Tank System

Consider the two-tank system (Quadros et al., 2020; Lopes et al., 2020b; Oliveira et al., 2022) described in Figure 5.4.

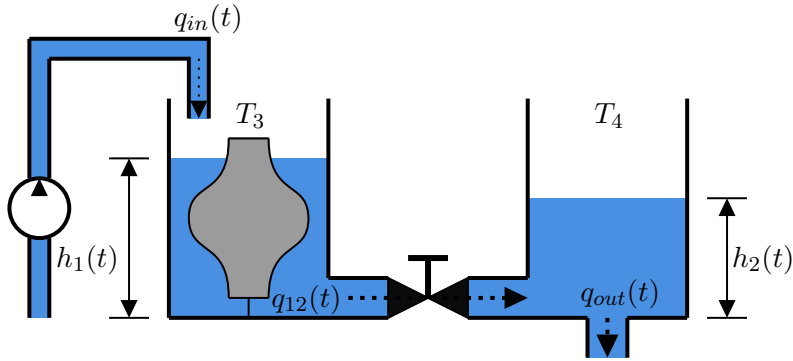


Figure 5.4: Schematic of the coupled tank system used for real-time experimentation. A pump motor recirculates water from tank T_3 to tank T_4 . Tank T_3 contains an internal structure, resulting in nonlinear dynamic behavior.

The system is modeled based on mass balance equations and Bernoulli's principle as follows:

$$\begin{aligned} \dot{h}_1(t) &= \frac{q_{in}(t)}{A_1(h_1(t))} - \frac{h_1(t) - h_2(t)}{R_{12}(h_1(t), h_2(t))A_1(h_1(t))}, \\ \dot{h}_2(t) &= \frac{h_1(t) - h_2(t)}{R_{12}(h_1(t), h_2(t))A_2} - \frac{q_{out}(t)}{A_2}, \end{aligned} \quad (5.34)$$

where $h_1(t)$ and $h_2(t)$ are the water levels of tanks T_3 and T_4 , respectively, the flow equations were experimentally obtained as:

$$q_{in}(t) = K_b u(t) \text{ cm}^3/\text{s},$$

where the input flow is controlled by the signal $\hat{u}(t)$, $K_b = 26.8934$, and

$$q_{out}(t) = 0.0041h_2(t)^3 - 0.4451h_2(t)^2 + 36.4733h_2(t) + 507.1889 \text{ cm}^3/\text{s},$$

representing the output flow. The flow between the tanks depends on the hydraulic resistance given by

$$R_{12}(h_1(t), h_2(t)) = 0.1192(h_1(t) - h_2(t))^3 - 2.45(h_1(t) - h_2(t))^2 + 60.2987(h_1(t) - h_2(t)) + 449.7407 \text{ s/cm}^2.$$

The cross-sectional area of tank T_4 is given by $A_2 = 3019 \text{ cm}^2$, whereas the area of tank T_3 is described by

$$A_1(h_1(t)) = \frac{3r}{5} \left(2.7r - \frac{\cos(2.5\pi((h_1(t) - 8) \times 10^{-2} - \psi))}{\omega\sqrt{2\pi}} \times e^{-\frac{((h_1(t) - 8) \times 10^{-2} - \psi)^2}{2\omega^2}} \right) \times 10^4 \text{ cm}^2,$$

with $r = 31 \text{ cm}$, $\omega = 0.55$, and $\psi = 0.4$. Therefore, a *quasi*-LPV system can be obtained as:

$$\begin{bmatrix} \dot{h}_1(t) \\ \dot{h}_2(t) \end{bmatrix} = \begin{bmatrix} -\rho_1(t)\rho_3(t) & \rho_1(t)\rho_3(t) \\ \frac{\rho_1(t)}{A_2} & \frac{-\rho_1(t) - \rho_2(t)}{A_2} \end{bmatrix} \begin{bmatrix} h_1(t) \\ h_2(t) \end{bmatrix} + \begin{bmatrix} K_b\rho_3(t) \\ 0 \end{bmatrix} \hat{u}(k), \quad (5.35)$$

where the *quasi*-LPV parameters are given by $\rho_1(t) = 1/R_{12}$, $\rho_2(t) = q_{out}/h_2(t)$, $\rho_3(t) = 1/A_1(h_1(t))$. The operating intervals are $32 \leq h_1 \leq 65 \text{ cm}$ and $20 \leq h_2 \leq 45 \text{ cm}$. Accordingly, the corresponding ranges for the functions $\rho_i(t)$ are given by:

$$\begin{aligned} 79.5402 &\leq \rho_1(t) \leq 407.9372, \\ 35.9926 &\leq \rho_2(t) \leq 54.5660, \text{ and} \\ 3.4478 \times 10^{-4} &\leq \rho_3(t) \leq 6.9159 \times 10^{-4}. \end{aligned} \quad (5.36)$$

Using the sector nonlinearity approach (Tanaka and Wang, 2001), the *quasi*-LPV model with $N = 8$ in Γ_8 can be rewritten.

Remark 5.5. Traditionally, to control such an industrial process, discrete-time models are considered to design a discrete-time controller under constant sampling intervals. For instance, as done in works such as Quadros et al. (2020); Lopes et al. (2020b); Oliveira et al. (2022) (and Chapter 3), the forward Euler method can be employed to derive a discrete-time model. By doing so, it can be observed that for a sampling time of $T_s \approx 17 \text{ s}$, the discretized model begins to exhibit vertices with eigenvalues with modulus greater than one, indicating unstable behavior that is not the case for the considered tank system. Thus, illustrating the nontrivial task of discretizing the nonlinear (or *quasi*-LPV) system model. Alternatively, sampled-data controllers can be designed directly from the continuous-time model, avoiding the need for time discretization and preserving the original system dynamics.

Robust Controller Design

Based on the operating intervals, an operating point with a control signal of $u^* = 50.69\%$, is selected, yielding tank levels of $h_1^* = 48.5 \text{ cm}$ and $h_2^* = 32.5 \text{ cm}$. Thus, the state vector is rewritten as $x(t) = [h_1(t) - h_1^* \quad h_2(t) - h_2^*]^\top$ and the control signal as $u(t) = \hat{u}(t) - u^*$. The validity domain of the *quasi*-LPV model is limited by $|x_1(t)| \leq 16.5 \text{ cm}$ and $|x_2(t)| \leq 12.5$

cm, according to the defined operating intervals. Consequently, the subset $\mathcal{R}_x$ in (5.30) can be characterized by

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \sigma = [16.5 \quad 12.5]^\top.$$

The objective is to control the level of tank T_3 from a given initial condition using sampling intervals $\eta_k \in [1, 30]$ s. Assuming that the derivative bounds of the time-varying parameters are unknown, the region of attraction is estimated by applying the optimization procedure (5.33) with parameter-independent matrices (Remark 5.4) and $\epsilon = 0.5$. The resulting value is $\gamma = 0.0067$, with the Lyapunov matrix

$$P = \begin{bmatrix} 0.0037 & -0.0003 \\ -0.0003 & 0.0064 \end{bmatrix},$$

and a robust control gain matrix $K = [-0.6684 \quad -0.0576]$. The estimated region of attraction

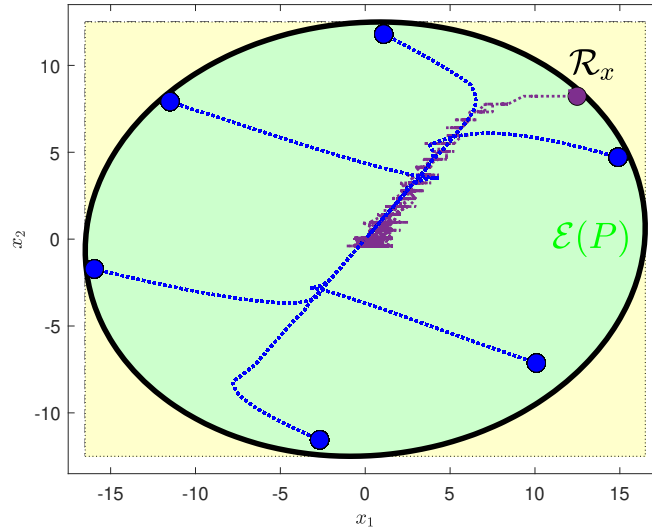


Figure 5.5: State-space of the region of attraction with simulated trajectories (blue dotted lines) and real-time experiment trajectory (purple dotted line).

is then described by the ellipsoidal set $\mathcal{E}(P) = \{x \in \mathbb{R}^n, x^\top P x \leq 1\}$, shown in Figure 5.5, which is entirely contained within the domain of validity $\mathcal{R}_x$ of the model. Several simulations are performed using the designed controller with random sampling intervals $\eta_k \in [1, 30]$ s, starting from initial conditions close to the border of the estimated region of attraction, and their corresponding trajectories (blue dotted lines). An initial condition $x(0) = [11.5 \quad 8.5]^\top$ is selected on the boundary of the region of attraction, highlighted by the purple dot. This corresponds to initial tank levels of $h_1(0) = 60$ cm and $h_2(0) = 41$ cm, respectively.

The designed controller is applied to the system from the selected initial condition under four different sampling patterns (random, constant at the maximum, increasing, and decreasing) within the interval $\eta_k \in [1, 30]$ s. The time-varying sampling intervals were imposed by software, emulating a network effect. The purple dotted line in Figure 5.5 shows the states of the real-time experiment with a random sampling interval converging to the origin.

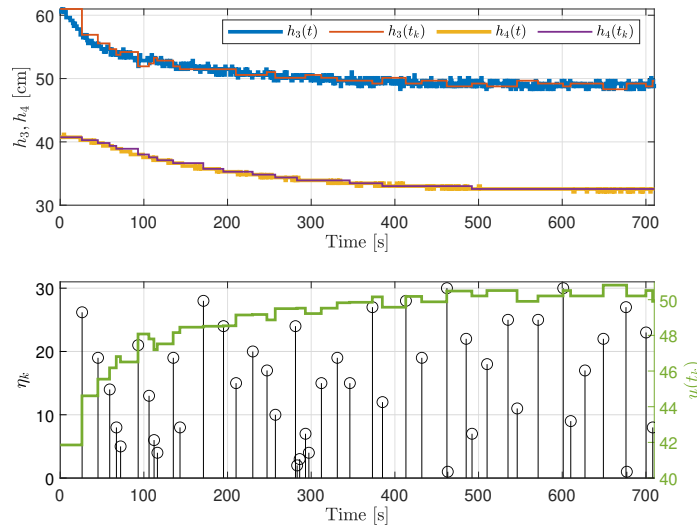


Figure 5.6: Real-time experiment with random time sampling interval (bottom), control signal (bottom), and states of the system (top).

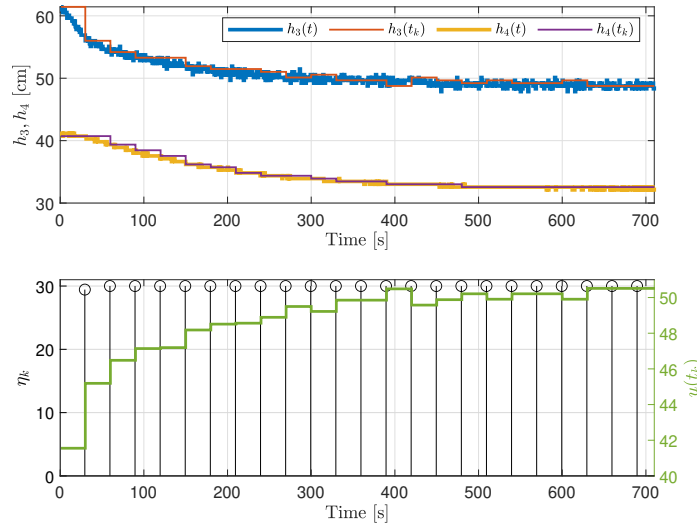


Figure 5.7: Real-time experiment with constant time sampling interval set at the maximum allowed, $\eta_k = \mathcal{T}_2 = 30$ s, (bottom), control signal (bottom), and states of the system (top).

Figures 5.6–5.9 show the tank levels during the experiments, where the values are measured at the sampling instants and held constant until the next sample (top plots), along with the corresponding control signals and sampling intervals (bottom plots). The results show that the proposed method successfully drives the tank levels asymptotically to the equilibrium point, even under varying sampling conditions. This set of real-time experiments suggests the effectiveness of the proposed approach in practical applications.

In contrast to traditional discrete-time designs, which become unstable for sampling times larger than $T_s \approx 17$ s (see Remark 5.5), the sampled-data approach remains effective even with sampling intervals up to 30 s, as shown in Figure 5.7. This emphasizes the advantage of the proposed method in preserving stability and performance under significantly larger and

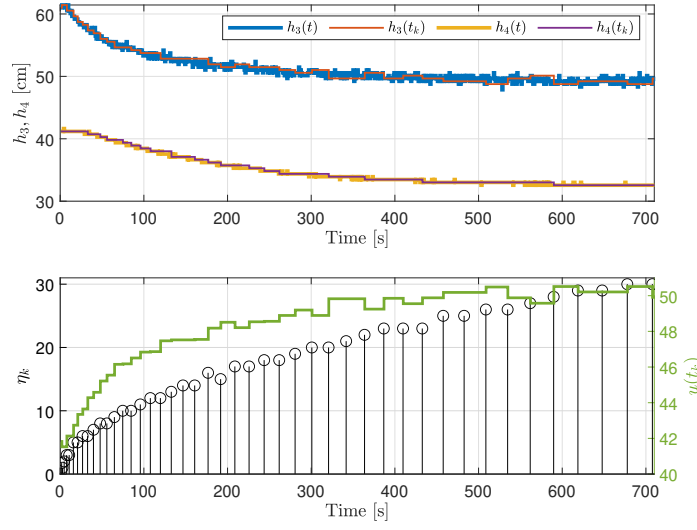


Figure 5.8: Real-time experiment with increasing time sampling interval, $\eta_{k+1} = \eta_k + 1$, $\eta_0 = 1$ s (bottom), control signal (bottom), and states of the system (top).

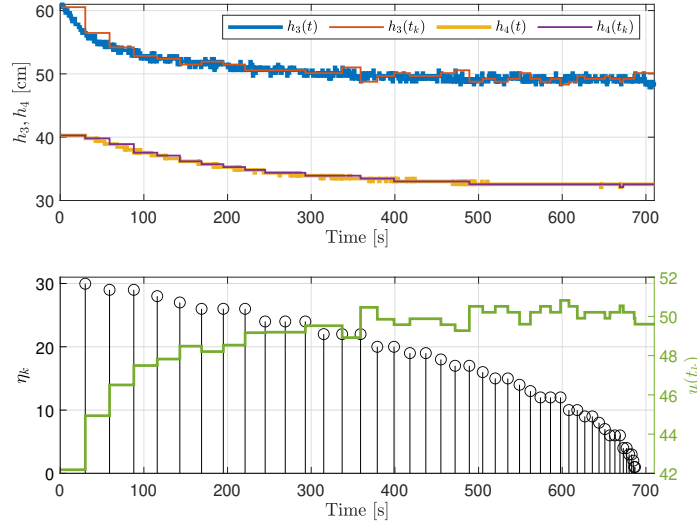


Figure 5.9: Real-time experiment with decreasing time sampling interval $\eta_{k+1} = \eta_k - 1$, $\eta_0 = 30$ s (bottom), control signal (bottom), and states of the system (top).

time-varying sampling intervals, which discrete-time approaches cannot guarantee.

5.3 Parameter-Dependent Stabilization of LPV Sampled-Data Systems via Quadratic Polynomial Conditions

This chapter concludes with an analysis of how parameter-dependent gain controllers, combined with stability ensured by the negativity of a quadratic polynomial, can reduce the conservatism of the proposed solutions. In this context, Problem 5.1 is investigated by applying the mean value theorem, presented in Section 5.1.1, together with the negativity lemmas introduced in Section 5.2.2. Therefore, consider the following theorem:

Theorem 5.4. *If there exists matrices $\mathbf{0} < \tilde{P}(\alpha(t)) \in \mathbb{S}^n$, $\mathbf{0} < \tilde{R} \in \mathbb{S}^n$, $\tilde{S}(\alpha(t)) \in \mathbb{S}^n$, $\tilde{Q}(\alpha(t)) \in \mathbb{R}^{n \times n}$, $\tilde{U} \in \mathbb{S}^{2n}$, $\tilde{L}(\alpha(t)) \in \mathbb{R}^{3n \times n}$, $\tilde{K}(\alpha(t)) \in \mathbb{R}^{n_u \times n}$, $\tilde{X} \in \mathbb{R}^{n \times n}$, and a scalar $\epsilon > 0$ satisfying*

$$\tau^2(t)\tilde{\Pi}_2 + \tau(t)\tilde{\Pi}_1(\alpha(t)) + \tilde{\Pi}_0(\alpha(t)) < 0, \forall \tau(t) \in [0, \mathcal{T}_i], \quad (5.37)$$

$\forall i = 1, 2$, with

$$\begin{aligned} \tilde{\Pi}_2 &= -\text{He}\{\bar{E}_1^\top \tilde{U} \bar{E}_2\}, \\ \tilde{\Pi}_1(\alpha(t)) &= \text{He}\left\{-E_3^\top \left(\tilde{Q}(\alpha(t))E_2 + \tilde{S}(\alpha(t))E_{12}\right) - E_{12}^\top \dot{\tilde{Q}}(\alpha(t))E_2 + \epsilon E_1^\top B(\alpha(t))\dot{\tilde{K}}(\alpha(t-c(t)))E_2 \right. \\ &\quad \left. + E_3^\top B(\alpha(t))\dot{\tilde{K}}(\alpha(t-c(t)))E_2\right\} - \mathcal{T}_i \tilde{\Pi}_2 - 2\bar{E}_1^\top \tilde{U} \bar{E}_1 - E_3^\top \tilde{R}E_3 - E_{12}^\top \dot{\tilde{S}}(\alpha(t))E_{12} \\ &\quad + \tilde{L}(\alpha(t))\tilde{R}^{-1}\tilde{L}^\top(\alpha(t)), \\ \tilde{\Pi}_0(\alpha(t)) &= \text{He}\left\{E_1^\top \tilde{P}(\alpha(t))E_3 - E_{12}^\top \tilde{Q}(\alpha(t))E_2 - \tilde{L}(\alpha(t))E_{12} + (\epsilon E_1^\top + E_3^\top)(-A(\alpha(t))\tilde{X}E_1 \right. \\ &\quad \left. - B(\alpha(t))\tilde{K}(\alpha(t))E_2 + \tilde{X}E_3) + \mathcal{T}_i E_3^\top (\tilde{Q}(\alpha(t))E_2 + \tilde{S}(\alpha(t))E_{12}) + \mathcal{T}_i E_{12}^\top \dot{\tilde{Q}}(\alpha(t))E_2\right\} \\ &\quad + \mathcal{T}_i (E_3^\top \tilde{R}E_3 + \bar{E}_1^\top \tilde{U} \bar{E}_1 + E_{12}^\top \dot{\tilde{S}}(\alpha(t))E_{12}) - E_{12}^\top \tilde{S}(\alpha(t))E_{12} + E_1^\top \dot{\tilde{P}}(\alpha(t))E_1, \end{aligned}$$

where $E_1 = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix}$, $E_2 = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \end{bmatrix}$, $E_3 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}$, $E_{12} = E_1 - E_2$, $\bar{E}_1 = \begin{bmatrix} E_1 & E_2 \end{bmatrix}^\top$, $\bar{E}_2 = \begin{bmatrix} E_3 & \mathbf{0} \end{bmatrix}^\top$. Then, system (5.1) is asymptotically stabilized by the sampled-data controller (5.3) where

$$K(\alpha(t_k)) = \tilde{K}(\alpha(t_k))\tilde{X}^{-1},$$

for any sampling sequences satisfying $0 < \mathcal{T}_1 \leq \eta_k \leq \mathcal{T}_2$.

Proof: The proof of this theorem is similar to that of Theorem 5.3, jointly with the application of the mean value theorem as demonstrated in the proof of Theorem 5.2. $\blacksquare$

5.3.1 Illustrative Example

Consider the model of a helicopter in vertical flight presented in Example 5.2.4, restated for the convenience of the reader. The state variables are the horizontal velocity ($x_1(t)$), vertical velocity ($x_2(t)$), pitch rate ($x_3(t)$), and pitch angle ($x_4(t)$). The control inputs are the collective pitch control ($u_1(t)$) and longitudinal cyclic pitch control ($u_2(t)$). The system matrices are given by:

$$A(\rho_1(t), \rho_2(t)) = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.0100 & 0.0024 & -4.0208 \\ 0.1002 & \rho_1(t) & -0.7070 & \rho_2(t) \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B(\rho_3(t)) = \begin{bmatrix} 0.4422 & 0.1761 \\ \rho_3(t) & -7.5922 \\ -5.5200 & 4.9900 \\ 0 & 0 \end{bmatrix},$$

with $\rho_1(t) \in [-0.0664, 0.5047]$, $\rho_2(t) \in [0.1198, 2.526]$, and $\rho_3(t) \in [0.9775, 5.112]$. The respective rates of variation satisfy $\dot{\rho}_1(t) \in [-0.15, 0.15]$, $\dot{\rho}_2(t) \in [-0.8, 0.8]$ and $\dot{\rho}_3(t) \in [-1.7, 1.7]$. Since the system has three time-varying parameters, a polytope with $N = 8$ is obtained in Γ_8 .

For a minimum sampling time of $\mathcal{T}_1 = 0.1$, Theorem 5.2, which employs a looped-functional yielding a linear function in $\tau(t)$ with a parameter-dependent controller, is compared with Theorems 5.3 (robust controller) and 5.4 (parameter-dependent controller), both based on looped-functionals resulting in a quadratic polynomial in $\tau(t)$, combined with Lemmas 5.2 and 5.3

Table 5.3: Maximum value of $\mathcal{T}_2$

Condition	Controller Gain Structure	$\mathcal{T}_2$
Theorem 5.2	$K(\alpha(t_k))$	0.954
Theorem 5.3 with Lemma 5.2	K	0.859
Theorem 5.3 with Lemma 5.3, $d = 5$	K	0.864
Theorem 5.4 with Lemma 5.2	$K(\alpha(t_k))$	0.919
Theorem 5.4 with Lemma 5.3, $d = 5$	$K(\alpha(t_k))$	0.965

to ensure negative definiteness. From Table 5.3, it can be observed that parameter-dependent controller gains allow a larger admissible sampling interval, highlighting the effectiveness of the proposed mean value theorem in mitigating the mismatch effect. Moreover, the quadratic polynomial approach proves to be effective when used with less conservative negative definiteness conditions, such as Lemma 5.3. Thus, combining the mean value theorem with negativity lemmas for quadratic polynomials provides a promising strategy to obtain less conservative sampling interval estimates while ensuring stability.

5.4 Final Considerations

This chapter presents a simple yet effective method based on the mean value theorem to address parameter mismatches between the controller and the plant during intersampling periods. A key advantage of the proposed approach is that it prevents mismatch terms from spreading across the parameter-dependent matrices in the controller synthesis conditions. Consequently, mismatch effects vanish at the sampling instants, enabling more accurate estimates of the maximum admissible sampling interval compared to recent literature. The numerical example illustrates the effectiveness of the method, showing improved results even when compared with more complex approaches. Overall, the findings highlight the potential of this proposal to enhance the design of sampled-data controllers for LPV systems within the input-delay framework, which can also be explored under alternative settings such as impulsive and hybrid approaches. Moreover, a looped-functional whose time derivative yields a quadratic polynomial was proposed and applied in the controller synthesis framework. The lemma based on Pólya's theorem was employed to ensure the negativity of second-degree functions, making it suitable for handling the derivative of the looped-functional. In addition, particular conditions to deal with *quasi*-LPV systems were introduced. Finally, real-time experiments on a nonlinear tank system with industrial actuators demonstrated the practical applicability of the proposed methods.

Sampled-Data Control of LPV Systems under Saturating Actuators

Josué,

Faz muito tempo que eu não mando uma carta para alguém.

Agora eu tô mandando esta carta pra você.

Você tem razão: seu pai ainda vai aparecer, e com certeza ele é tudo aquilo que você diz que ele é.

Eu lembro do meu pai me levando na locomotiva que ele dirigia. Ele deixou eu, uma menininha, dar o apito do trem a viagem inteira.

Quando você estiver cruzando as estradas no seu caminhão enorme, espero que você lembre que fui eu a primeira pessoa a te fazer botar a mão no volante.

Também vai ser melhor você ficar aí com seus irmãos.

Você merece muito, muito mais do que eu tenho pra te dar.

No dia que você quiser lembrar de mim, dá uma olhada no retratinho que a gente tirou junto.

Eu digo isso porque tenho medo que um dia você também me esqueça.

Tenho saudade do meu pai.

Tenho saudade de tudo.

Dora

Central do Brasil (1998)
dir. WALTER SALLES

This chapter presents the parameter-dependent aperiodic sampled-data state feedback controller design for LPV systems with actuators subject to magnitude and rate saturation, utilizing LMIs. The proposed method integrates the looped-functional approach and a parameter-dependent generalized sector condition. The local stabilization is verified through a definite negativeness lemma for second-order matrix polynomials. The proposed conditions can be simplified to recover a robust controller design whenever the time-varying parameter is unavailable. Two numerical examples demonstrate the effectiveness of the proposed method, highlighting less conservative stability conditions compared to existing approaches. Moreover, an experimental validation is performed on a sampled-data control of a nonlinear tank-level system, considering actuator saturation both in magnitude and rate, demonstrating the practical applicability of the proposed approach.

6.1 Problem Formulation

Consider a continuous-time plant modeled as a polytopic LPV system defined as:

$$\dot{x}_s(t) = A(\alpha(t))x_s(t) + B(\alpha(t))\Phi(u(t)), \quad (6.1)$$

where $x_s \in \mathbb{R}^n$ is the state of the system vector and $\Phi(u(t)) \in \mathbb{R}^{n_u}$ is the input delivered by the actuator vector. The time-varying parameter vector is supposed to be online available and given by $\alpha(t) \in \Gamma_N$ where

$$\Gamma_N = \{\delta \in \mathbb{R}^N : \sum_{i=1}^N \delta_i = 1, \delta_i \geq 0, i = 1, \dots, N\}.$$

It is assumed that $\alpha_i(t) \in \mathcal{C}^1$ and $|\dot{\alpha}_i(t)| \leq \phi_i, \forall t \geq 0$, and $i = 1, \dots, N$. The matrices $A(\alpha(t)) \in \mathbb{R}^{n \times n}$ and $B(\alpha(t)) \in \mathbb{R}^{n \times n_u}$ have a polytopic structure with N known vertices given by

$$\begin{bmatrix} A(\alpha(t)) & B(\alpha(t)) \end{bmatrix} = \sum_{i=1}^N \alpha_i(t) \begin{bmatrix} A_i & B_i \end{bmatrix}.$$

Concerning realistic process control conditions, the actuator of the plant is considered to be saturated within symmetrical magnitude and variation bounds. Thus, it follows that

$$\text{sat}_{\mathbf{M}}(u_{(\ell)}(t)) = \text{sign}(u_{(\ell)}(t)) \min\{|u_{(\ell)}(t)|, \mu_{\mathbf{M}(\ell)}\}$$

and

$$\text{sat}_{\mathbf{R}}(u_{(\ell)}(t)) = \text{sign}(u_{(\ell)}(t)) \min\{|u_{(\ell)}(t)|, \mu_{\mathbf{R}(\ell)}\},$$

for $\ell = 1, \dots, n_u$, where the vectors $\mu_{\mathbf{M}} \in \mathbb{R}^{n_u}$ and $\mu_{\mathbf{R}} \in \mathbb{R}^{n_u}$ with positive elements define the saturation limits and indexes $\mathbf{M}$ and $\mathbf{R}$ concerns magnitude and rate, respectively. The first-order model, known as the position-type feedback model with speed limitation ([Palmeira et al., 2016](#)), is considered, where the magnitude and rate saturation are embedded as

$$\dot{x}_{a(\ell)}(t) = \text{sat}_{\mathbf{R}(\ell)} \left(\lambda_{(\ell)} \text{sat}_{\mathbf{M}(\ell)} \left(u_{(\ell)}(t) \right) - \lambda_{(\ell)} x_{a(\ell)}(t) \right), \quad (6.2)$$

for $\ell = 1, \dots, n_u$, with $x_a \equiv \Phi(u(t)) \in \mathbb{R}^{n_u}$ and $u \in \mathbb{R}^{n_u}$ stand for the state of the actuator state and (linear) control input, respectively. The parameter $\lambda_{(\ell)} > 0$ regards the actuator dynamics, usually chosen as $\lambda \rightarrow \infty$.

An LPV sampled-data state feedback controller is considered, defined as

$$u(t) = K_s(\alpha(t_k))x_s(t_k) + K_a(\alpha(t_k))x_a(t_k), \quad (6.3)$$

where t_k concerns the aperiodic sampling instants, such that $t_{k+1} > t_k$ and $\cup_{k \in \mathbb{N}} [t_k, t_{k+1}) = [0, +\infty)$, while the control signal is held constant between sampling instants by means of a zero-order hold (ZOH). The interval between two consecutive sampling instants is denoted by $\eta_k = t_{k+1} - t_k$, which varies but remains constrained by two positive scalars $\mathcal{T}_1 \leq \mathcal{T}_2$, such that

$0 < \mathcal{T}_1 \leq \eta_k \leq \mathcal{T}_2$. Similar to the plant model, the controller matrices $K_s(\alpha(t_k)) \in \mathbb{R}^{n_u \times n}$ and $K_a(\alpha(t_k)) \in \mathbb{R}^{n_u \times n_u}$ have polytopic structure

$$\begin{bmatrix} K_s(\alpha(t_k)) & K_a(\alpha(t_k)) \end{bmatrix} = \sum_{i=1}^N \alpha_i(t_k) \begin{bmatrix} K_{si} & K_{ai} \end{bmatrix}.$$

Considering $\Phi(u(t)) = x_a(t)$, with (6.1), (6.2), and (6.3), the closed-loop system can be obtained as

$$\begin{aligned} \dot{x}_s(t) &= A(\alpha(t))x_s(t) + B(\alpha(t))x_a(t), \\ \dot{x}_a(t) &= \text{sat}_R(\Lambda \text{sat}_M(K_s(\alpha(t_k))x_s(t_k) + K_a(\alpha(t_k))x_a(t_k)) - \Lambda x_a(t)), \forall t \in [t_k, t_{k+1}), \end{aligned} \quad (6.4)$$

where $\Lambda \in \mathbb{R}^{n_u \times n_u}$ is a diagonal matrix composed by $\Lambda_{(\ell, \ell)} = \lambda_{(\ell)}$, with $-\lambda_{(\ell)}$ being the eigenvalues of the actuators.

A schematic of the sampled-data state feedback control topology implemented with magnitude and rate saturating actuators is shown in Fig. 6.1, where the dotted lines indicate the digital signals computed at each sampling instant t_k .

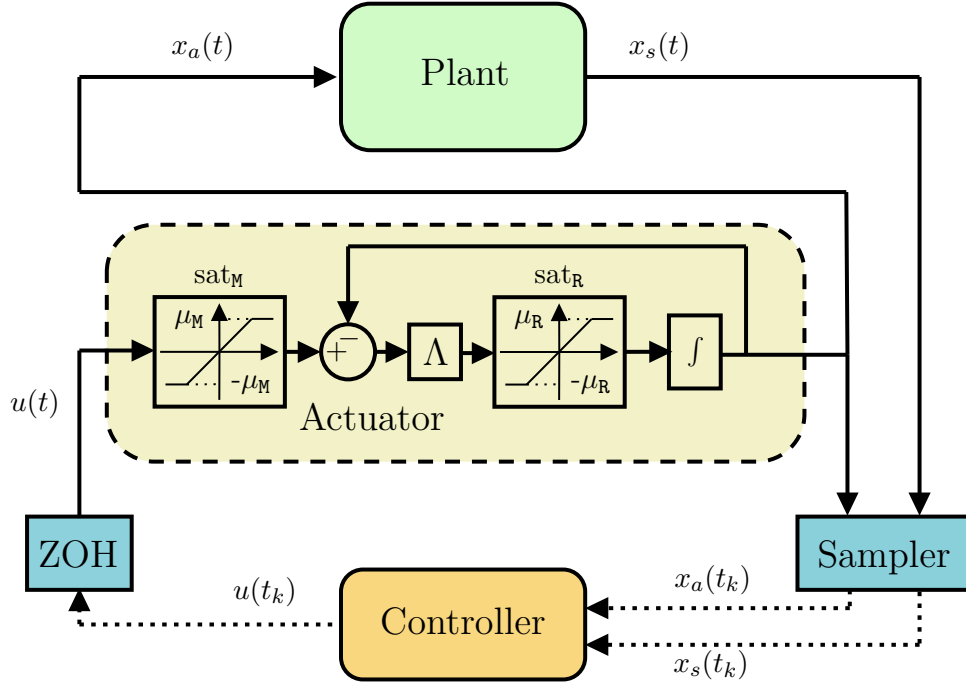


Figure 6.1: Control topology of sampled-data state feedback with actuators under magnitude and rate saturation.

As stated by Oliveira et al. (2024) and Palmeira et al. (2016), the use of model (6.2) to describe magnitude and rate saturating actuators can be seen as a particular case of nested saturation (Tarbouriech et al., 2006). Also, the closed-loop system (6.4) can be rewritten as

$$\begin{aligned} \dot{x}(t) &= \mathbb{A}_2(\alpha(t))x(t) + \mathbb{B}_2 \text{sat}_R(\mathbb{A}_1 x(t) + \mathbb{B}_1 \text{sat}_M(u(t))), \\ u(t) &= \mathbb{K}(\alpha(t_k)x(t_k)), \forall t \in [t_k, t_{k+1}), \end{aligned} \quad (6.5)$$

with the augmented state vector $x = \begin{bmatrix} x_s^\top & x_a^\top \end{bmatrix}^\top$, $x \in \mathbb{R}^{n_x}$, $n_x = n + n_u$, with

$$\mathbb{A}_2(\alpha(t)) = \begin{bmatrix} A(\alpha(t)) & B(\alpha(t)) \\ \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\mathbb{A}_1 = \begin{bmatrix} \mathbf{0} & -\Lambda \end{bmatrix}, \mathbb{B}_2 = \begin{bmatrix} \mathbf{0} & \mathbf{I} \end{bmatrix}^\top, \mathbb{B}_1 = \Lambda, \text{ and } \mathbb{K}(\alpha(t_k)) = \begin{bmatrix} K_s(\alpha(t_k)) & K_a(\alpha(t_k)) \end{bmatrix}.$$

Due to the presence of saturating actuators, there exists a set of initial conditions, referred to as the region of attraction $\mathcal{R}_a \subseteq \mathbb{R}^{n_x}$, whose corresponding trajectories asymptotically converge to the origin of the closed-loop system (6.5). However, analytically computing $\mathcal{R}_a$ is a challenging task, as it may be non-convex or unbounded in certain directions (Tarbouriech et al., 2011). Therefore, an estimate is considered as

$$\mathcal{E}(R_0) = \{x \in \mathbb{R}^{n_x}; x^\top R_0 x \leq 1\} \subseteq \mathcal{R}_a \subseteq \mathbb{R}^{n_x},$$

taking into account the effects of saturating actuators and aperiodic samplings.

The objective of this chapter is to develop convex conditions for the synthesis of LPV sampled-data state feedback controllers that ensure the local asymptotic stability of the system (6.5), under magnitude and rate saturating actuators and aperiodic samplings with $\eta_k \in [\mathcal{T}_1, \mathcal{T}_2]$. Hence, the following problems are defined:

Problem 6.1. *Given the LPV system described by (6.1), a set of initial conditions $\mathcal{E}(R_0)$, and the lower bound on the sampling time $\mathcal{T}_1$, determine the parameter-dependent controller gains according to the control law (6.3), such that the maximum allowable upper bound of the sampling interval, $\mathcal{T}_2$, is maximized and all trajectories starting within $\mathcal{E}(R_0)$ asymptotically converge to the origin, without leaving such a region.*

Problem 6.2. *Given the LPV system described by (6.1) and sampling interval limits $[\mathcal{T}_1, \mathcal{T}_2]$, determine the parameter-dependent controller gains according to the control law (6.3), such that the estimate of the region of attraction $\mathcal{E}(R_0)$ is maximized.*

6.2 Auxiliary Results

The sampled-data controlled system can be represented as a time-varying delay system by applying the input-delay approach (Fridman, 2010), where $\tau(t) \in [0, \eta_k]$ is the time-varying delay, such that $\dot{\tau}(t) = 1$ and $t_k = t - \tau(t)$, $\forall t \in [t_k, t_{k+1})$. Thus, the closed-loop system (6.5) can be rewritten as

$$\dot{x}(t) = \mathbb{A}_2(\alpha(t))x(t) + \mathbb{B}_2 \text{sat}_R(\mathbb{A}_1 x(t) + \mathbb{B}_1 \text{sat}_M(\mathbb{K}(\alpha(t - \tau(t)))x(t - \tau(t)))). \quad (6.6)$$

To analyze the local stability of (6.6), the closed-loop system is reformulated as a Lur'e-type representation to handle the nonlinearities associated with magnitude and rate saturation. Thus, using decentralized dead-zone functions

$$\Psi_M = \text{sat}_M(\mathbb{K}(\alpha(t - \tau(t)))x(t - \tau(t))) - \mathbb{K}(\alpha(t - \tau(t)))x(t - \tau(t)),$$

$$\Psi_R = \text{sat}_R(\mathbb{A}_1 x(t) + \mathbb{B}_1 \text{sat}_M(\mathbb{K}(\alpha(t - \tau(t)))x(t - \tau(t)))) - (\mathbb{A}_1 x(t) + \mathbb{B}_1 \text{sat}_M(\mathbb{K}(\alpha(t - \tau(t)))x(t - \tau(t)))),$$

(6.6) can be rewritten as

$$\dot{x}(t) = (\mathbb{A}_2(\alpha(t)) + \mathbb{B}_2 \mathbb{A}_1)x(t) + \mathbb{B}_2 \mathbb{B}_1 \mathbb{K}(\alpha(t - \tau(t)))x(t - \tau(t)) + \mathbb{B}_2 \mathbb{B}_1 \Psi_M + \mathbb{B}_2 \Psi_R. \quad (6.7)$$

To solve Problems 6.1 and 6.2, some adaptations of known results from the literature are required, which are presented in the next three lemmas, with the main differences summarized as follows: Lemma 6.1 includes in Tarbouriech et al. (2011) parameter-dependent variables to handle magnitude and rate saturation; Lemma 6.2 includes both parameter-dependent variables and the generalized sector conditions extending Seuret (2012); Palmeira et al. (2016); and Lemma 6.3 extends result in Park and Park (2020) to LPV sampled-data framework to handle second-order polynomials in $\tau(t)$. At last, Lemma 6.4, presented in Chapter 5, is a result that plays a key role in this contribution to reduce the conservatism due to the parameter mismatch between the system and the controller.

Lemma 6.1. *Consider the sets*

$$\mathbb{S}_M(\mu_M) = \{x(t - \tau(t)) \in \mathbb{R}^{n_x} : \\ |(\mathbb{K}_{(\ell)}(\alpha(t - \tau(t))) - G_{0(\ell)}(\alpha(t - \tau(t))))x(t - \tau(t))| \leq \mu_{M(\ell)}, \ell = 1, \dots, n_u\}$$

and

$$\mathbb{S}_R(\mu_R) = \{(x(t), x(t - \tau(t)), \Psi_M) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} : \\ |\Xi_{1(\ell)}x(t) + \Xi_{2(\ell)}x(t - \tau(t)) + \Xi_{3(\ell)}\Psi_M| \leq \mu_{R(\ell)}, \ell = 1, \dots, n_u\},$$

with matrices $G_0(\alpha(t - \tau(t)))$, $G_1(\alpha(t))$, $G_2(\alpha(t - \tau(t))) \in \mathbb{R}^{n_u \times n_x}$, and $G_3 \in \mathbb{R}^{n_x \times n_x}$ where $\Xi_1 = \mathbb{A}_1 - G_1(\alpha(t))$, $\Xi_2 = \mathbb{B}_1\mathbb{K}(\alpha(t - \tau(t))) - G_2(\alpha(t - \tau(t)))$, and $\Xi_3 = \mathbb{B}_1 - G_3$. If $x(t - \tau(t)) \in \mathbb{S}_M(\mu_M)$ and $(x(t), x(t - \tau(t)), \Psi_M) \in \mathbb{S}_R(\mu_R)$, then the dead-zone functions Ψ_M and Ψ_R , satisfy

$$\Psi_M^\top T_M(\Psi_M + G_0(\alpha(t - \tau(t)))x(t - \tau(t))) \leq 0, \quad (6.8)$$

$$\Psi_R^\top T_R(\Psi_R + G_1(\alpha(t))x(t) + G_2(\alpha(t - \tau(t)))x(t - \tau(t)) + G_3\Psi_M) \leq 0. \quad (6.9)$$

with diagonal matrices $\mathbf{0} < T_M \in \mathbb{S}^{n_u}$ and $\mathbf{0} < T_R \in \mathbb{S}^{n_u}$.

Proof: The proof follows the application of the generalized sector condition in Lemma 1 of Palmeira et al. (2016) and Lemma 3.1, with parameter-dependent variables selected based on the time domain of the elements in the set that each variable multiplies. ■

Lemma 6.2. *Consider a function $V : \mathbb{K} \times \Gamma_N \rightarrow \mathbb{R}_+$, a continuous and differentiable parameter-dependent function $\mathcal{V}_0 : \mathbb{K} \times \Gamma_N \times [0, \eta_k] \times [\eta_1, \eta_2] \rightarrow \mathbb{R}$ satisfying*

$$\kappa_1 \|x\|^2 \leq V(x, \alpha) \leq \kappa_2 \|x\|^2, \quad (6.10)$$

$$\begin{aligned} \mathcal{V}_0(x, \alpha, 0, \eta_k) &= \mathcal{V}_0(x, \alpha, \eta_k, \eta_k) = 0, \\ \mathcal{V}_0(x, \alpha, \tau, \eta_k) &> 0, \end{aligned} \quad (6.11)$$

for all $x \in \mathbb{R}^{n_x}$, $\alpha \in \Gamma_N$, $\tau \in [0, \eta_k]$, and $\eta_k \in [\mathcal{T}_1, \mathcal{T}_2]$. Considering $\mathcal{W}(x, \alpha, \tau, \eta_k) = V(x, \alpha) + \mathcal{V}_0(x, \alpha, \tau, \eta_k)$ and

$$\dot{\mathcal{W}}(x, \alpha, \tau, \eta_k) = \frac{d}{dt}(V(x, \alpha) + \mathcal{V}_0(x, \alpha, \tau, \eta_k)),$$

suppose there exist matrices $G_0(\alpha(t - \tau(t)))$, $G_1(\alpha(t))$, $G_2(\alpha(t - \tau(t))) \in \mathbb{R}^{n_u \times n_x}$, $G_3 \in \mathbb{R}^{n_x \times n_x}$, defining Ξ_1 , Ξ_2 , and Ξ_3 given in Lemma 6.1, positive definite diagonal matrices T_M , $T_R \in \mathbb{R}^{n_u \times n_u}$, and positive scalars κ_1 , κ_2 , and κ_3 , such that the following inequalities hold

$$\begin{aligned} & x^\top(t - \tau(t))(\mathbb{K}_{(\ell)}(\alpha(t - \tau(t))) - G_{0(\ell)}(\alpha(t - \tau(t))))^\top \\ & \times (\mathbb{K}_{(\ell)}(\alpha(t - \tau(t))) - G_{0(\ell)}(\alpha(t - \tau(t))))x(t - \tau(t)) \\ & \leq \mu_{M(\ell)}^2 V(x(t - \tau(t)), \alpha(t - \tau(t))), \end{aligned} \quad (6.12)$$

$$\begin{bmatrix} x(t) \\ x(t - \tau(t)) \\ \Psi_M \end{bmatrix}^\top \begin{bmatrix} \Xi_1^\top(\ell) \\ \Xi_2^\top(\ell) \\ \Xi_3^\top(\ell) \end{bmatrix} \begin{bmatrix} \Xi_1(\ell) & \Xi_2(\ell) & \Xi_3(\ell) \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \tau(t)) \\ \Psi_M \end{bmatrix} \leq \mu_{R(r)}^2 V(x(t), \alpha(t)), \quad (6.13)$$

for $\ell = 1, \dots, n_u$,

$$\begin{aligned} & \dot{W}(x, \alpha, \tau, \eta_k) - 2\Psi_M^\top T_M(\Psi_M + G_0(\alpha(t - \tau(t)))x(t - \tau(t))) - 2\Psi_R^\top T_R \\ & \times (\Psi_R + G_1(\alpha(t))x(t) + G_2(\alpha(t - \tau(t)))x(t - \tau(t)) + G_3\Psi_M) < -\kappa_3 \left\| \begin{bmatrix} x(t) \\ x(t - \tau(t)) \end{bmatrix} \right\|^2, \end{aligned} \quad (6.14)$$

for $\forall(k, \alpha(t), \tau(t), \eta_k) \in \mathbb{N} \times \Gamma_N \times [0, \eta_k] \times [\mathcal{T}_1, \mathcal{T}_2]$. Then, any initial condition $x(0)$ belonging to the set:

$$\mathcal{L}_V = \{x \in \mathbb{R}^{n_x}, \alpha \in \Gamma_N; V(x, \alpha) \leq 1\}, \quad (6.15)$$

ensures that:

i) the increment of the Lyapunov function $V(x(t), \alpha(t))$ between two consecutive sampling instants is strictly negative, i.e., it satisfies

$$\Delta V(x(t_k), \alpha(t_k)) = V(x(t_{k+1}), \alpha(t_{k+1})) - V(x(t_k), \alpha(t_k)) < -\kappa_3 \left\| \begin{bmatrix} x(t) \\ x(t - \tau(t)) \end{bmatrix} \right\|^2;$$

ii) the trajectories of the closed-loop system (6.6) with aperiodic samplings $\eta_k \in [\mathcal{T}_1, \mathcal{T}_2]$ converge asymptotically to the origin without leaving $\mathcal{L}_V$.

Proof: Assuming that $x(0) \in \mathcal{L}_V$, taking (6.12) and (6.13), hence, for $\ell = 1, \dots, n_u$,

$$|(\mathbb{K}_{(\ell)}(\alpha(t - \tau(t))) - G_{0(\ell)}(\alpha(t - \tau(t))))x(t - \tau(t))| \leq \mu_{M(\ell)}$$

implies $x(t - \tau(t)) \in \mathbb{S}_M(\mu_M)$ and

$$|(\Xi_1)_{(\ell)}x(t) + (\Xi_2)_{(\ell)}x(t - \tau(t)) + (\Xi_3)_{(\ell)}\Psi_M| \leq \mu_{R(\ell)}$$

implies $(x(t), x(t - \tau(t)), \Psi_M) \in \mathbb{S}_R(\mu_R)$. Then, (6.8), (6.9), and (6.14) yield

$$\dot{W}(x, \alpha, \tau, \eta_k) < -\kappa_3 \left\| \begin{bmatrix} x(t) \\ x(t - \tau(t)) \end{bmatrix} \right\|^2 \leq 0.$$

The result can be demonstrated thereafter using reasoning similar to Theorem 1 in [Palmeira et al. \(2016\)](#) and [Palmeira et al. \(2021\)](#). ■

Remark 6.1. Following the approach in *Palmeira et al. (2016)*, where rate saturation is considered for LTI systems, the definition in (6.11) is necessary to ensure that the actuator state trajectories remain within $\mathbb{S}_R(\mu_R)$ throughout the sampling intervals. Therefore, $\mathcal{W}$ provides an upper bound for $V(x, \alpha)$ between sampling instants while being equal to it at each sample.

The following lemmas, originally presented in Chapters 4 and 5, are restated for the convenience of the reader.

Lemma 6.3. Consider a quadratic parameter-dependent function $f(y) = \mathcal{A}_2(\alpha(t))y^2 + \mathcal{A}_1(\alpha(t))y + \mathcal{A}_0(\alpha(t))$, where $\mathcal{A}_2(\alpha(t))$, $\mathcal{A}_1(\alpha(t))$, $\mathcal{A}_0(\alpha(t)) \in \mathbb{R}^{p \times p}$. If there exists a matrix $M(\alpha(t)) \in \mathbb{R}^{p \times p}$ satisfying $M(\alpha(t)) + M(\alpha(t))^\top \geq \mathbf{0}$, such that

$$\begin{bmatrix} f(\underline{h}) & \frac{1}{2}\mathcal{A}_1(\alpha(t)) + \underline{h}\mathcal{A}_2(\alpha(t)) + (\bar{h} - \underline{h})M(\alpha(t)) \\ \star & \mathcal{A}_2(\alpha(t)) - \text{He}\{M(\alpha(t))\} \end{bmatrix} < \mathbf{0}, \quad (6.16)$$

then it follows that $f(y) < 0$, $\forall y \in [\underline{h}, \bar{h}]$.

Lemma 6.4. Consider a parameter-dependent matrix $Y(\alpha(t - \tau(t))) = \sum_{i=1}^N \alpha_i(t - \tau(t))Y_i$. If $\alpha_i(t) \in \mathcal{C}^1$ and $|\dot{\alpha}_i(t)| \leq \phi_i$, for $i = 1, \dots, N$, thus it is equivalent to rewrite

$$Y(\alpha(t - \tau(t))) = Y(\alpha(t)) - \tau(t)\dot{Y}(\alpha(t - c(t))). \quad (6.17)$$

where $c(t) \in [0, \tau(t)]$ and $\dot{Y}(\alpha(t - c(t))) = \sum_{i=1}^N \dot{\alpha}_i(t - c_i(t))Y_i$.

Proof: From the mean value theorem, if $f : [a, b] \rightarrow \mathbb{R}$ is a continuous function on $[a, b]$. Then, there exists $c \in [a, b]$ such that $\int_a^b f(s)ds = (b - a)f(c)$. Therefore, with $\alpha_i(t) \in \mathcal{C}^1$ there exists a $c_i(t) \in [0, \tau(t)]$ such that

$$\alpha_i(t) - \alpha_i(t - \tau(t)) = \int_{t-\tau(t)}^t \dot{\alpha}_i(s)ds = \tau(t)\dot{\alpha}_i(t - c_i(t)),$$

for $i = 1, \dots, N$. Hence, any parameter-dependent matrix $Y(\alpha(t - \tau(t))) = \sum_{i=1}^N \alpha_i(t - \tau(t))Y_i$ can be rewritten as

$$Y(\alpha(t - \tau(t))) = Y(\alpha(t)) - \sum_{i=1}^N \int_{t-\tau(t)}^t \dot{\alpha}_i(s)ds Y_i = Y(\alpha(t)) - \tau(t)\dot{Y}(\alpha(t - c(t))).$$

■

6.3 Main Results

This section presents convex conditions for designing a controller (6.3) and addressing Problems 6.1 and 6.2.

Consider the auxiliary matrices $E_1 = [\mathbf{I} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0}]$, $E_2 = [\mathbf{0} \ \mathbf{I} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0}]$, $E_3 = [\mathbf{0} \ \mathbf{0} \ \mathbf{I} \ \mathbf{0} \ \mathbf{0}]$, $E_4 = [\mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{I} \ \mathbf{0}]$, $E_5 = [\mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{I}]$, $E_{12} = E_1 - E_2$, $\bar{E}_{24} = [E_2^\top \ E_4^\top]^\top$, $\bar{E}_{12} = [E_{12}^\top \ \mathbf{0}]^\top$, $\bar{E}_3 = [E_3^\top \ \mathbf{0}]^\top$, and $\bar{E}_4 = [\mathbf{0} \ E_4^\top]^\top$.

Theorem 6.1. *Suppose there exists matrices $\mathbf{0} < \tilde{P}(\alpha(t)) \in \mathbb{S}^{n_x}$, $\mathbf{0} < \tilde{R} \in \mathbb{S}^{n_x}$, $\mathbf{0} < \tilde{S}(\alpha(t)) \in \mathbb{S}^{n_x}$, $\mathbf{0} < \tilde{U} \in \mathbb{S}^{(n_x+n_u)}$, $\tilde{L}(\alpha(t)) \in \mathbb{R}^{(3n_x+2n_u) \times n_x}$, $\tilde{\mathbb{K}}(\alpha(t)) \in \mathbb{R}^{n_u \times n_x}$, $\tilde{G}_0(\alpha(t)) \in \mathbb{R}^{n_u \times n_x}$, $\tilde{G}_1(\alpha(t)) \in \mathbb{R}^{n_u \times n_x}$, $\tilde{X} \in \mathbb{R}^{n_x \times n_x}$, $\tilde{M}_i(\alpha(t)) \in \mathbb{R}^{(3n_x+2n_u) \times (3n_x+2n_u)}$, diagonal matrices $\mathbf{0} < \tilde{T}_M \in \mathbb{S}^{n_u}$, $\mathbf{0} < \tilde{T}_R \in \mathbb{S}^{n_u}$, and a scalar $\epsilon > 0$, such that, for $i = 1, 2$ and $\ell = 1, \dots, n_u$,*

$$\begin{bmatrix} \tilde{\Pi}_0(\alpha(t)) & \frac{1}{2}\tilde{\Pi}_1(\alpha(t)) + \mathcal{T}_i\tilde{M}_i(\alpha(t)) & \mathbf{0} \\ \star & \tilde{\Pi}_2 - \text{He}\{\tilde{M}_i(\alpha(t))\} & \tilde{L}(\alpha(t)) \\ \star & \star & -\mathcal{T}_i\tilde{R} \end{bmatrix} < \mathbf{0}, \quad (6.18)$$

$$\begin{bmatrix} \text{He}\{\tilde{M}_i(\alpha(t))\} & \tilde{L}(\alpha(t)) \\ \star & \mathcal{T}_i\tilde{R} \end{bmatrix} > \mathbf{0}, \quad (6.19)$$

$$\begin{bmatrix} \tilde{P}(\alpha(t)) & (\tilde{\mathbb{K}}_{(\ell)}(\alpha(t)) - \tilde{G}_{0(\ell)}(\alpha(t)))^\top \\ \star & \mu_{M(\ell)}^2 \end{bmatrix} \geq \mathbf{0}, \quad (6.20)$$

$$\begin{bmatrix} \tilde{P}(\alpha(t)) & (\tilde{\mathbb{K}}_{(\ell)}(\alpha(t)) - \mathcal{T}_i\tilde{\mathbb{K}}_{(\ell)}(\alpha(t-c(t))))^\top - (\tilde{G}_{0(\ell)}(\alpha(t)) - \mathcal{T}_i\tilde{G}_{0(\ell)}(\alpha(t-c(t))))^\top \\ \star & \mu_{M(\ell)}^2 \end{bmatrix} \geq \mathbf{0}, \quad (6.21)$$

$$\begin{bmatrix} \tilde{P}(\alpha(t)) & ((\mathbb{A}_1\tilde{X})_{(\ell)} - \tilde{G}_{1(\ell)}(\alpha(t)))^\top \\ \star & \mu_{R(\ell)}^2 \end{bmatrix} \geq \mathbf{0}, \quad (6.22)$$

with

$$\begin{aligned} \tilde{\Pi}_2 &= -\text{He}\left\{\bar{E}_{12}^\top \tilde{U} \bar{E}_3 + \bar{E}_3^\top \tilde{U} \bar{E}_4\right\}, \\ \tilde{\Pi}_1(\alpha(t)) &= -2\left(\bar{E}_{12}^\top \tilde{U} \bar{E}_{12} + \bar{E}_4^\top \tilde{U} \bar{E}_4 + \text{He}\left\{\bar{E}_{12}^\top \tilde{U} \bar{E}_4\right\}\right) - \mathcal{T}_i\tilde{\Pi}_2 + \text{He}\left\{-E_3^\top \tilde{S}(\alpha(t))E_{12}\right. \\ &\quad \left.+ E_4^\top \tilde{G}_0(\alpha(t-c(t)))E_2 + E_5^\top \mathbb{B}_1\tilde{\mathbb{K}}(\alpha(t-c(t)))E_2 + (\epsilon E_1^\top + E_3^\top)(\mathbb{B}_2\mathbb{B}_1\tilde{\mathbb{K}}(\alpha(t-c(t)))E_2)\right\} \\ &\quad - E_3^\top \tilde{R}E_3 - E_{12}^\top \dot{\tilde{S}}(\alpha(t))E_{12} \\ \tilde{\Pi}_0(\alpha(t)) &= \text{He}\left\{E_1^\top \tilde{P}(\alpha(t))E_3 - \tilde{L}(\alpha(t))E_{12} - E_4^\top \tilde{G}_0(\alpha(t))E_2 - E_5^\top \tilde{G}_1(\alpha(t))E_1\right. \\ &\quad \left.- E_5^\top \mathbb{B}_1\tilde{\mathbb{K}}(\alpha(t))E_2 - E_5^\top \tilde{T}_M E_4 + (\epsilon E_1^\top + E_3^\top)((-\mathbb{A}_2(\alpha(t)) - \mathbb{B}_2\mathbb{A}_1)\tilde{X}E_1 - \mathbb{B}_2\mathbb{B}_1\tilde{\mathbb{K}}(\alpha(t))E_2\right. \\ &\quad \left.+ \tilde{X}E_3 - \mathbb{B}_2\mathbb{B}_1\tilde{T}_M E_4 - \mathbb{B}_2\tilde{T}_R E_5) + \mathcal{T}_i(E_3^\top \tilde{S}(\alpha(t))E_{12} + \bar{E}_{12}^\top \tilde{U} \bar{E}_4)\right\} + \mathcal{T}_i(E_3^\top \tilde{R}E_3 + E_{12}^\top \dot{\tilde{S}}(\alpha(t))E_{12} \\ &\quad \left.+ \bar{E}_{12}^\top \tilde{U} \bar{E}_{12} + \bar{E}_4^\top \tilde{U} \bar{E}_4) - E_{12}^\top \tilde{S}(\alpha(t))E_{12} + E_1^\top \dot{\tilde{P}}(\alpha(t))E_1 - 2E_4^\top \tilde{T}_M E_4 - 2E_5^\top \tilde{T}_R E_5. \end{aligned}$$

Then, for any initial condition $x(0)$ belonging to the set $\mathcal{E}(P(\alpha(t))) = \{x \in \mathbb{R}^{n_x}, x^\top P(\alpha(t))x \leq 1\}$, where $P(\alpha(t)) = \tilde{X}^{-\top} P(\alpha(t)) \tilde{X}^{-1}$, the respective trajectories of the LPV sampled-data system (6.5) with $\mathbb{K}(\alpha(t_k)) = \tilde{\mathbb{K}}(\alpha(t_k))\tilde{X}^{-1}$ converge asymptotically to the origin.

Proof: Consider $\mathcal{W}(x(t), \alpha(t), \tau(t), \eta_k) = V(x(t), \alpha(t)) + \mathcal{V}_0(x(t), \alpha(t), \tau(t), \eta_k)$ where

$$V(x(t), \alpha(t)) = x^\top(t)P(\alpha(t))x(t),$$

$$\begin{aligned} \mathcal{V}_0(x(t), \alpha(t), \tau(t), \eta_k) &= (\eta_k - \tau(t))\left[(x(t) - x(t - \tau(t)))^\top S(\alpha(t))(x(t) - x(t - \tau(t)))\right. \\ &\quad \left.+ \tau\zeta^\top U\zeta + \int_{t-\tau(t)}^t \dot{x}^\top(s)R\dot{x}(s)ds\right], \end{aligned}$$

with $\zeta = [(x(t) - x(t - \tau(t)))^\top \Psi_M^\top]^\top$, positive definite matrices $P(\alpha(t)), R \in \mathbb{S}^{n_x}$, $U \in \mathbb{S}^{(n_x+n_u)}$, and $S(\alpha(t)) \in \mathbb{S}^{n_x}$. Therefore, $V(x(t), \alpha(t))$ and $\mathcal{V}_0(x(t), \alpha(t), \tau(t), \eta_k)$ satisfy (6.10)

and (6.11). $\mathcal{E}(P(\alpha(t)))$ can be exactly computed as the intersection of a finite number of ellipsoids, given by $\bigcap_{\alpha(t) \in \Gamma} \mathcal{E}(P(\alpha(t))) = \bigcap_{i \in 1, \dots, N} \mathcal{E}(P_i)$ (see [Figueiredo et al. \(2021\)](#) for the general formulation assuming homogeneous parameter dependency).

The time derivative of $\mathcal{W}(x(t), \alpha(t), \tau(t), \eta_k)$ yields

$$\begin{aligned} \dot{\mathcal{W}}(x(t), \alpha(t), \tau(t), \eta_k) &= 2\dot{x}^\top(t)P(\alpha(t))x(t) + x^\top(t)\dot{P}(\alpha(t))x(t) \\ &- (x(t) - x(t - \tau(t)))^\top S(\alpha(t))(x(t) - x(t - \tau(t))) + (\eta_k - \tau(t))2\dot{x}^\top(t)S(\alpha(t))(x(t) - x(t - \tau(t))) \\ &+ (\eta_k - \tau(t))(x(t) - x(t - \tau(t)))^\top \dot{S}(\alpha(t))(x(t) - x(t - \tau(t))) - \tau(t)\dot{\zeta}^\top U\zeta + (\eta_k - \tau(t))\dot{\zeta}^\top U\zeta \\ &+ (\eta_k - \tau(t))\tau(t)2\dot{\zeta}^\top U\zeta + (\eta_k - \tau(t))\dot{x}^\top(t)R\dot{x}(t) - \int_{t-\tau(t)}^t \dot{x}^\top(s)R\dot{x}(s)ds, \end{aligned}$$

with $\dot{\zeta} = \begin{bmatrix} \dot{x}^\top(t) & \mathbf{0}^\top \end{bmatrix}^\top$. The last term of $\dot{\mathcal{W}}(x(t), \alpha(t), \tau(t), \eta_k)$ is overbounded using the Jensen's inequality as

$$- \int_{t-\tau(t)}^t \dot{x}^\top(s)R\dot{x}(s)ds \leq -2\xi^\top L(\alpha(t))(x(t) - x(t - \tau(t))) + \tau(t)\xi^\top L(\alpha(t))R^{-1}L^\top(\alpha(t))\xi,$$

with $L(\alpha(t)) \in \mathbb{R}^{(3n_x+2n_u) \times n_x}$ and $\xi = \begin{bmatrix} x^\top(t) & x^\top(t - \tau(t)) & \dot{x}^\top(t) & \Psi_M^\top & \Psi_R^\top \end{bmatrix}^\top$. From (6.7), let $\mathcal{B} = \begin{bmatrix} -\mathbb{A}_2(\alpha(t)) - \mathbb{B}_2\mathbb{A}_1 & -\mathbb{B}_2\mathbb{B}_1\mathbb{K}(\alpha(t - \tau(t))) & \mathbf{I} & -\mathbb{B}_2\mathbb{B}_1 & -\mathbb{B}_2 \end{bmatrix}$, such that $\mathcal{B}\xi = \mathbf{0}$ holds. Let $\mathcal{X} = X^\top(\epsilon E_1^\top + E_3^\top)$ and $X \in \mathbb{R}^{n_x \times n_x}$. From (6.14), with the application of the Finsler's Lemma with $G_2(\alpha(t - \tau(t))) = \mathbb{B}_1\mathbb{K}(\alpha(t - \tau(t)))$ and $G_3 = \mathbb{B}_1$, it follows that

$$\begin{aligned} \dot{\mathcal{W}}(x(t), \alpha(t), \tau(t), \eta_k) &- 2\Psi_M^\top T_M(\Psi_M + G_0(\alpha(t - \tau(t)))x(t - \tau(t))) - 2\Psi_R^\top T_R(\Psi_R + G_{1\alpha}x(t) \\ &+ \mathbb{B}_1\mathbb{K}(\alpha(t - \tau(t)))x(t - \tau(t)) + \mathbb{B}_1\Psi_M) \leq \xi^\top \mathcal{Q}(\tau(t))\xi \end{aligned}$$

with

$$\begin{aligned} \mathcal{Q}(\tau(t)) &= \Pi_2\tau^2(t) + (\Pi_1(\alpha(t)) + L(\alpha(t))R^{-1}L^\top(\alpha(t)))\tau(t) + \Pi_0(\alpha(t)), \\ \Pi_2 &= -\mathbf{He} \left\{ \bar{E}_{12}^\top U \bar{E}_3 + \bar{E}_3^\top U \bar{E}_4 \right\}, \\ \Pi_1(\alpha(t)) &= -2 \left(\bar{E}_{12}^\top U \bar{E}_{12} + \bar{E}_4^\top U \bar{E}_4 + \mathbf{He} \left\{ \bar{E}_{12}^\top U \bar{E}_4 \right\} \right) - \eta_k \Pi_2 + \mathbf{He} \left\{ -\bar{E}_3^\top S(\alpha(t))E_{12} \right\} \\ &\quad - E_3^\top R E_3 - E_{12}^\top \dot{S}(\alpha(t))E_{12}, \\ \tilde{\Pi}_0(\alpha(t)) &= \mathbf{He} \left\{ E_1^\top P(\alpha(t))E_3 - L(\alpha(t))E_{12} - E_4^\top T_M G_0(\alpha(t - \tau(t)))E_2 - E_5^\top T_R G_1(\alpha(t))E_1 \right. \\ &\quad - E_5^\top T_R \mathbb{B}_1 \mathbb{K}(\alpha(t - \tau(t)))E_2 - E_5^\top T_R \mathbb{B}_1 E_4 + (\epsilon X E_1^\top + X E_3^\top)((-\mathbb{A}_2(\alpha(t)) - \mathbb{B}_2\mathbb{A}_1)E_1 \\ &\quad - \mathbb{B}_2\mathbb{B}_1\mathbb{K}(\alpha(t))E_2 + E_3 - \mathbb{B}_2\mathbb{B}_1 E_4 - \mathbb{B}_2 E_5) + \eta_k(E_3^\top S(\alpha(t))E_{12} + \bar{E}_{12}^\top U \bar{E}_4) \left. \right\} + \eta_k(E_3^\top R E_3 \\ &\quad + E_{12}^\top \dot{S}(\alpha(t))E_{12} + \bar{E}_{12}^\top U \bar{E}_{12} + \bar{E}_4^\top U \bar{E}_4 - E_{12}^\top S(\alpha(t))E_{12} + E_1^\top \dot{P}(\alpha(t))E_1 \\ &\quad - 2E_4^\top T_M E_4 - 2E_5^\top T_R E_5. \end{aligned}$$

Block (3,3) of $\tilde{\Pi}_0(\alpha(t))$ yields $\tilde{X} + \tilde{X}^\top < \mathbf{0}$, ensuring the non-singularity of $\tilde{X}$. Assuming $\tilde{X} = X^{-1}$, $\tilde{T}_M = T_M^{-1}$, $\tilde{T}_R = T_R^{-1}$ and multiplying $\mathcal{Q}(\tau(t))$ on the left by $\text{diag} \{ \tilde{X}^\top, \tilde{X}^\top, \tilde{X}^\top, \tilde{T}_M, \tilde{T}_R \}$

and on the right by its transpose, the following variable changes are applied:

$$\begin{aligned}\tilde{P}(\alpha(t)) &= \tilde{X}^\top P(\alpha(t)) \tilde{X}, & \tilde{S}(\alpha(t)) &= \tilde{X}^\top S(\alpha(t)) \tilde{X}, \\ \tilde{U} &= \text{diag}\{\tilde{X}^\top, \tilde{T}_M^\top\} U \text{diag}\{\tilde{X}, \tilde{T}_M\}, & \tilde{R} &= \tilde{X}^\top R \tilde{X}, \\ \tilde{L}(\alpha(t)) &= \text{diag}\{\tilde{X}^\top, \tilde{X}^\top, \tilde{X}^\top, \tilde{T}_M, \tilde{T}_R\} L(\alpha(t)) \tilde{X}, & \tilde{\mathbb{K}}(\alpha(t - \tau(t))) &= \mathbb{K}(\alpha(t - \tau(t))) \tilde{X}, \\ \tilde{G}_0(\alpha(t - \tau(t))) &= G_0(\alpha(t - \tau(t))) \tilde{X}, & \tilde{G}_1(\alpha(t)) &= G_1(\alpha(t)) \tilde{X}.\end{aligned}$$

Applying Lemma 6.4 on $\mathbb{K}(\alpha(t - \tau(t)))$ and $\tilde{G}_0(\alpha(t - \tau(t)))$, condition (6.14) can be satisfied with

$$\tilde{\Pi}_2 \tau^2(t) + \left(\tilde{\Pi}_1(\alpha(t)) + \tilde{L}(\alpha(t)) \tilde{R}^{-1} \tilde{L}^\top(\alpha(t))^\top \right) \tau(t) + \tilde{\Pi}_0(\alpha(t)) < \mathbf{0}.$$

To ensure the negativity of the second-order function, Lemma 6.3 is applied with

$$M(\alpha(t)) = -\frac{1}{2\eta_k} \tilde{L}(\alpha(t)) \tilde{R}^{-1} \tilde{L}^\top(\alpha(t)) + \bar{M}(\alpha(t)),$$

following the steps of Park and Park (2020), for $\tau(t) \in [0, \eta_k]$, and $\eta_k \in [\mathcal{T}_1, \mathcal{T}_2]$ resulting in

$$\begin{bmatrix} \tilde{\Pi}_0(\alpha(t)) & \frac{1}{2} \tilde{\Pi}_1(\alpha(t)) + \mathcal{T}_i \bar{M}_i(\alpha(t)) \\ \star & \tilde{\Pi}_2 - \text{He}\{\bar{M}_i(\alpha(t))\} + \frac{1}{\mathcal{T}_i} L(\alpha(t)) \tilde{R}^{-1} \tilde{L}(\alpha(t))^\top \end{bmatrix} < \mathbf{0}.$$

Finally, a Schur complement is performed on this last inequality obtaining (6.18), as well in $\text{He}\{\bar{M}_i(\alpha(t))\} > \mathbf{0}$ to get (6.19). Then, the stability conditions of Lemma 6.2 are satisfied for $\tau(t) \in [0, \eta_k]$ and $\eta_k \in [\mathcal{T}_1, \mathcal{T}_2]$.

To conclude the proof, it is necessary to ensure that (6.12) and (6.13) hold. The reverse transformation of Lemma 6.4 is applied in (6.20) (for $\tau(t) = 0$) and (6.21) (for $\tau(t) = \eta_k$, with $\eta_k \in [\mathcal{T}_1, \mathcal{T}_2]$), and (6.20) (with $\tilde{\mathbb{K}}(\alpha(t - \tau(t)))$ and $\tilde{G}_0(\alpha(t - \tau(t)))$) is pre- and post-multiplied by $\text{diag}\{\tilde{X}^{-\top}, \mathbf{I}\}^\top$ and its transpose, respectively. By performing a Schur complement and pre- and post-multiplying by $x^\top(t - \tau(t))$ and $x(t - \tau(t))$, it follows that

$$\begin{aligned}x^\top(t - \tau(t)) & (P(\alpha(t)) - (\mathbb{K}_{(\ell)}(\alpha(t - \tau(t))) - G_{0(\ell)}(\alpha(t - \tau(t))))^\top \mu_{\mathbf{M}(\ell)}^{-2} \\ & \times (\mathbb{K}_{(\ell)}(\alpha(t - \tau(t))) - G_{0(\ell)}(\alpha(t - \tau(t)))) x(t - \tau(t)) \geq 0,\end{aligned}$$

verifying (6.12) and consequently $\mathcal{E}(P(\alpha(t))) \subseteq \mathbb{S}_{\mathbf{M}}(\mu_{\mathbf{M}})$. Similarly, (6.22) is pre- and post-multiplied by $\text{diag}\{\tilde{X}^{-\top}, \mathbf{I}\}^\top$ and its transpose, the Schur complement is taken, and the result is pre- and post-multiplied by $x^\top(t)$ and $x(t)$, yielding

$$x^\top(t) (P(\alpha(t)) - (\mathbb{A}_{1(\ell)} - G_{1(\ell)}(\alpha(t)))^\top \mu_{\mathbf{R}(\ell)}^{-2} (\mathbb{A}_{1(\ell)} - G_{1(\ell)}(\alpha(t)))) x(t) \geq 0,$$

which verifies (6.13), with $G_2(\alpha(t - \tau(t))) = \mathbb{B}_1 \mathbb{K}(\alpha(t - \tau(t)))$ and $G_3 = \mathbb{B}_1$, and consequently $\mathcal{E}(P(\alpha(t))) \subseteq \mathbb{S}_{\mathbf{R}}(\mu_{\mathbf{R}})$. This concludes the proof. $\blacksquare$

Remark 6.2. The number of scalar variables in Theorem 6.1 is

$$\nu_N = 2.5n_u + n_x + Nn_x + n_u n_x + 8Nn_u^2 + 22Nn_x^2 + 0.5n_u^2 + 2n_x^2 + 29Nn_u n_x.$$

When the parameters $\alpha(t)$ are not available for feedback, robust controllers are obtained by setting $\tilde{\mathbb{K}}(\alpha(t)) = \tilde{\mathbb{K}}$ and $\tilde{\mathbb{K}}(\alpha(t - c(t))) = \mathbf{0}$, and all parameter-dependent matrices $Y(\alpha(t)) = Y$, yielding the corresponding derivative ones as zero in Theorem 6.1. This simpler implementation has less computational cost, with ν_1 , but more conservative results.

6.3.1 Optimization Procedures

Maximization of $\mathcal{T}_2$: To address Problem 6.1, given $\mathcal{T}_1$, the control law (6.3) is determined to maximize $\mathcal{T}_2$ such that $\forall x(0) \in \mathcal{E}(R_0)$, the respective trajectories of the closed-loop system (6.6) converge asymptotically towards the origin. Considering the ellipsoidal set $\mathcal{E}(R_0) \subseteq \mathcal{R}_a$, ensuring to $R_0 \geq P(\alpha(t))$, with a given R_0 and

$$\begin{bmatrix} R_0 & \mathbf{I} \\ \star & -P(\alpha(t)) - \text{He}\{\tilde{X}\} \end{bmatrix} > \mathbf{0}. \quad (6.23)$$

Thus, consider the following procedure:

$$\mathcal{O}_1 : \begin{cases} \max & \mathcal{T}_2 \\ \text{subject to:} & (6.18) - (6.23). \end{cases} \quad (6.24)$$

To maximize $\mathcal{T}_2$, the optimization procedure (6.24) can be performed by checking the feasibility of LMIs (6.18)-(6.23) for given values of $\mathcal{T}_2$ and ϵ using a grid search.

Maximization of the estimate of the region of attraction: To address Problem 6.2, for a given sampling interval $[\mathcal{T}_1, \mathcal{T}_2]$, the control law (6.3) is determined to maximize the set $\mathcal{E}(P(\alpha(t)))$ through the following procedure:

$$\mathcal{O}_2 : \begin{cases} \min & \gamma \\ \text{subject to:} & (6.18) - (6.23) \text{ with } R_0 = \gamma \mathbf{I}. \end{cases} \quad (6.25)$$

As in the optimization procedure (6.24), a grid search in ϵ is performed, checking the feasibility of LMIs (6.18)-(6.23).

Remark 6.3. As Chapter 5, the ROLMIP parser (*Agulhari et al., 2019*) is used to derive a finite set of LMIs from the parameter-dependent conditions and their derivatives, and also to represent a parameter-dependent system in a convex formulation, which is augmented according to (6.5).

6.4 Illustrative Examples

Example 1: Consider the LPV system borrowed from *Ramezanifar et al. (2012)*:

$$\dot{x}_s(t) = \begin{bmatrix} 2\rho(t) & 1.1 + \rho(t) \\ -2.2 + \rho(t) & -3.3 + 0.1\rho(t) \end{bmatrix} x_s(t) + \begin{bmatrix} 2\rho(t) \\ 0.1 + \rho(t) \end{bmatrix} v(t),$$

with $|\rho(t)| \leq 1$, $|\dot{\rho}(t)| \leq 0.2$, $\Lambda = 20$, limits of magnitude and rate saturation are given by $\mu_M = 1$ and $\mu_R = 1$, and a minimum sampling interval bound $\mathcal{T}_1 = 0.01$ s. Considering $\mathcal{T}_2 = 0.1$ s and applying the optimization procedure $\mathcal{O}_2$ with $\epsilon = 1.5$, the result is $\gamma = 55.85$, controller gains $\mathbb{K}_1 = \begin{bmatrix} -0.0078 & -0.0084 & 0.9065 \end{bmatrix}$ and $\mathbb{K}_2 = \begin{bmatrix} -0.1941 & -0.0875 & 0.7128 \end{bmatrix}$, and an estimate region of attraction $\mathcal{L}_V = \mathcal{E}(P_1) \cap \mathcal{E}(P_2)$ yields the cut with $x_a = 0$ shown in Figure 6.2, where projections of the trajectories for several initial conditions are included. As expected, the trajectories (black lines) starting within $\mathcal{L}_V$ (purple region) converge asymptotically to the origin, and trajectories (red lines) initiating reasonably close to the boundaries of $\mathcal{L}_V$ diverge.

A simpler controller implementation, for instance when $\rho(t)$ is not available for feedback, can be provided with a robust controller following the directions in Remark 6.2, achieving more conservative results. In this case, the maximum feasible limit is $\mathcal{T}_2 = 0.078$ s. For $\mathcal{T}_2 = 0.05$ s, the region obtained is bounded by the yellow dashed line. The number of decision variables are $\nu_1 = 323$ (robust case) and $\nu_2 = 619$ (LPV case).

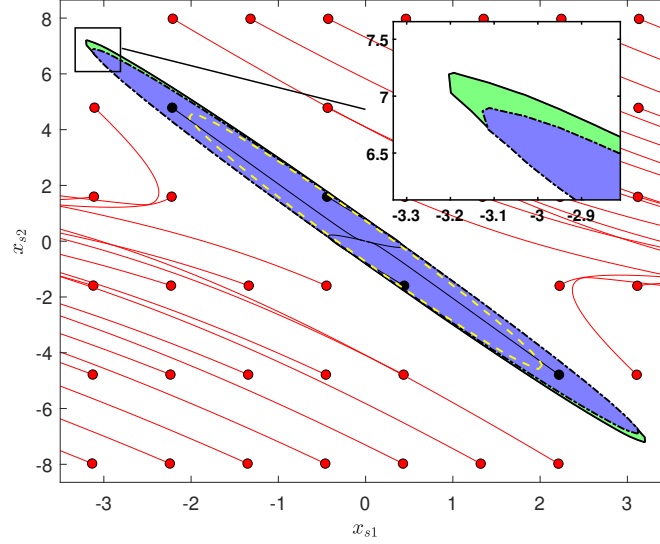


Figure 6.2: Cut of the estimate region of attraction $\mathcal{L}_V$ at $x_a = 0$ obtained with optimization procedure $\mathcal{O}_2$ and several trajectories projections.

To compare conditions of 3.1 with [Palmeira et al. \(2016\)](#), the time-invariant case with $\rho(t) = 0$ is considered. Using the optimization procedure $\mathcal{O}_1$ with $R_0 = 1.5\mathbf{I}$, assuming all matrix variables are parameter-independent, an upper limit of the sampling interval of $\mathcal{T}_2 = 0.205$ s is found. Conversely, [Palmeira et al. \(2016, Theorem 3\)](#) achieves $\mathcal{T}_2 = 0.176$ s, highlighting that the proposed functional used in 3.1 leads to less conservative solutions.

Example 2: Consider the uncertain helicopter model presented in [Chesi et al. \(2003, Ex. 5\)](#), with the system matrices given by:

$$A(\rho_1(t), \rho_2(t)) = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.0100 & 0.0024 & -4.0208 \\ 0.1002 & \rho_1(t) & -0.7070 & \rho_2(t) \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B(\rho_3(t)) = \begin{bmatrix} 0.4422 & 0.1761 \\ \rho_3(t) & -7.5922 \\ -5.5200 & 4.9900 \\ 0 & 0 \end{bmatrix},$$

with $|\rho_1(t)| \leq 0.2192$, $|\rho_2(t)| \leq 1.2031$, and $|\rho_3(t)| \leq 2.0673$, with $\Lambda = \text{diag}\{20, 20\}$, saturation limits $\mu_M = \begin{bmatrix} 4 & 4 \end{bmatrix}$ and $\mu_R = \begin{bmatrix} 1 & 1 \end{bmatrix}$, and a minimum sampling interval bound $\eta_1 = 0.001$ s.

The closed-loop augmented and written as (6.5) results in a sixth-order uncertain system with eight vertices and two control inputs. Assuming that the time-varying parameters are unknown, a robust controller is obtained using the change of variables in Remark 6.2. With $\mathcal{O}_1$, the maximum sampling interval bound is $\mathcal{T}_2 = 0.101$ s. Assuming $\mathcal{T}_2 = 0.05$ s and applying $\mathcal{O}_2$ with $\epsilon = 1.7$, the result is $\gamma = 5.9245$, yielding a safe ball with radius 0.41 in the state space. Figure 6.3 presents the closed-loop system simulation with the robust controller obtained for aperiodic samplings $\eta_k \in [0.001, 0.05]$ s, and initial conditions belonging to the region of

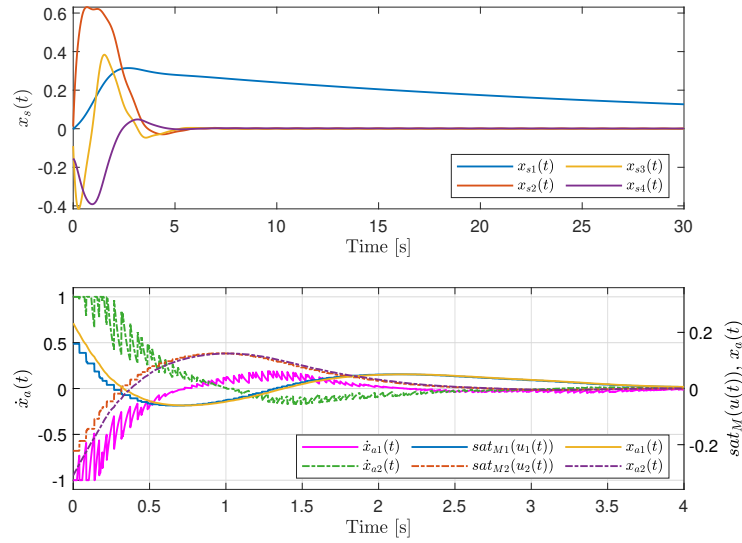


Figure 6.3: Top plot: states of the system, $x_s(t)$. Bottom plot: state of the actuator dynamics $\dot{x}_a(t)$ on left scale, state of the actuator $x_a(t)$, and control signal magnitude saturation function $\text{sat}_M(u(t))$ on right.

attraction. The top plot displays the evolution of the state of the system, $x_s(t)$, while the bottom plot shows the signals $\dot{x}_a(t)$, $x_a(t)$, and $\text{sat}_M(u(t))$ of both actuators. Despite the rate saturation of the actuator reaching their limits at the beginning of the simulation, the system remains stable, converging asymptotically to the origin. In case the LPV parameters are available with $|\dot{\rho}_1(t)| \leq 0.15$, $|\dot{\rho}_2(t)| \leq 0.8$, and $|\dot{\rho}_3(t)| \leq 1.7$, Theorem 6.1 yields a maximum $\mathcal{T}_2 = 0.114$ s. Similar to the robust case, with $\mathcal{T}_2 = 0.05$ s, $\mathcal{O}_2$ achieves $\gamma = 2.3183$: a safe ball of radius 0.6567, which is $\approx 60\%$ greater than the robust case. Such an increase in the safe ball occurs at the expense of increasing the decision variable numbers from $\nu_1 = 1275$ (robust gain) to $\nu_8 = 9521$ (LPV gain). Note that conditions in [Palmeira et al. \(2016\)](#) cannot be used in this case due to the LPV system.

6.5 Real-Time Experimental Validation with Saturating Actuators

Consider the nonlinear tank system described in Session 5.2.5, where the nonlinear model is given by (5.34) and the *quasi*-LPV model is described in (5.35). As discussed in Session 5.2.5, the domain of validity of the *quasi*-LPV model must be ensured in the stability conditions using LMI (5.31). Therefore, the region of attraction of a *quasi*-LPV model can be maximized with the following procedure:

$$\mathcal{O}_3 : \begin{cases} \min & \gamma \\ \text{subject to:} & (5.31), (6.18) - (6.23), \text{ with } R_0 = \gamma \mathbf{I}. \end{cases} \quad (6.26)$$

From the defined operating intervals, the operating point corresponding to a control input of $u^* = 50.69\%$ is selected, resulting in steady-state tank levels of $h_1^* = 48.5$ cm and $h_2^* = 32.5$

cm, and the state of actuator $x_a^* = 50.69\%$. Accordingly, the state vector is redefined as

$$x(t) = \begin{bmatrix} h_1(t) - h_1^* & h_2(t) - h_2^* & x_a(t) - x_a^* \end{bmatrix}^\top,$$

while the control signal is expressed as $u(t) = \hat{u}(t) - u^*$. The validity domain of the *quasi*-LPV model is then restricted to $|x_1(t)| \leq 16.5$ cm and $|x_2(t)| \leq 12.5$ cm, in accordance with the operating intervals. Consequently, the subset $\mathcal{R}_x$ in (5.30) is defined by

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \sigma = \begin{bmatrix} 16.5 & 12.5 \end{bmatrix}^\top.$$

Assuming the actuator constraints in magnitude and rate as $\mu_M = 15$ and $\mu_R = 1$, respectively, the actuator can vary from $\pm 15\%$ around the operation point u^* and with a limited variation of $\pm 1\%/s$. The actuator parameter is set as $\Lambda = 1$. The derivative bounds of the time-varying parameters are obtained experimentally as $\dot{\rho}_1 \leq |0.01|$, $\dot{\rho}_2 \leq |0.0417|$, and $\dot{\rho}_3 \leq |0.1671|$. Therefore, to control the level of tank T_3 for a given initial condition with aperiodic sampling intervals as $\eta_k \in [1, 30]$ s, the region of attraction is estimated with the optimization procedure $\mathcal{O}_3$ with $\epsilon = 0.4$, resulting in $\gamma = 0.0072$, and controller gains

$$\begin{aligned} K_1 = K_2 &= \begin{bmatrix} -0.2616 & -0.0094 & 0.0199 \end{bmatrix}, & K_3 = K_4 &= \begin{bmatrix} -0.2531 & -0.0094 & 0.0211 \end{bmatrix}, \\ K_5 = K_6 &= \begin{bmatrix} -0.1748 & -0.0083 & 0.0162 \end{bmatrix}, & K_7 = K_8 &= \begin{bmatrix} -0.1662 & -0.0083 & 0.0175 \end{bmatrix}. \end{aligned} \quad (6.27)$$

The estimated region of attraction is described by $\bigcap_{i \in 1, \dots, 8} \mathcal{E}(P_i)$ resulting in the cut with $x_a = 0$ shown in Figure 6.4. The initial condition highlighted by the purple dot, $x_p(0) = \begin{bmatrix} -11.5 & -9.7 \end{bmatrix}^\top$ is selected, corresponding to initial tank levels of $h_1(0) = 37.0$ cm and $h_2(0) = 22.8$ cm.

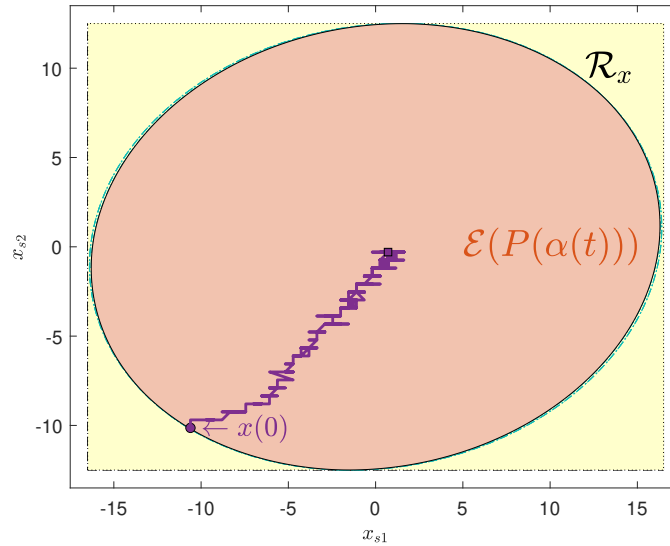


Figure 6.4: Cut of the estimate region of attraction $\mathcal{E}(P(\alpha(t)))$ at $x_a = 0$ obtained with optimization procedure $\mathcal{O}_3$, and real-time experiment trajectory (purple line).

The parameter-dependent controller gains (6.27) are applied to the system from the selected initial condition under two different sampling patterns, increasing and constant at the maximum, within the interval $\eta_k \in [1, 30]$ s, where the sampling intervals were enforced by software

simulating the network effects. The purple line in Figure 6.4 presents the states of the real-time experiment with samplings of 30 s converging to the origin without leaving the region of attraction. It is worth noting that the use of a parameter-dependent Lyapunov function is not particularly effective in this case for obtaining larger estimates of the region of attraction, since all sets obtained at the vertices of $P(\alpha(t))$ are practically the same.

Figures 6.5–6.7 show the real-time experiment with constant sampling intervals set at the maximum ($\eta_k = \mathcal{T}_2 = 30$ s). Figure 6.5 presents the tank levels during the experiment and the measurements, used for state-feedback control, executed at each sampling instant, which are held constant until the next sample. The bottom plot presents the sampling intervals pattern during the experiment.

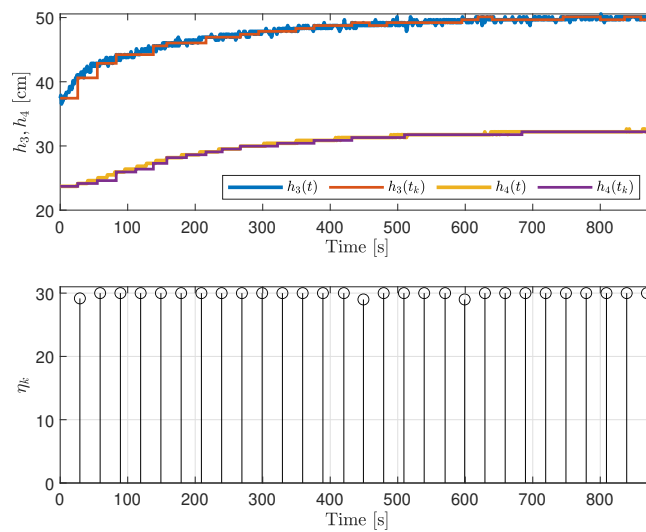


Figure 6.5: Real-time experiment under saturating actuator with constant sampling interval fixed at $\eta_k = \mathcal{T}_2 = 30$ s. Tank levels and corresponding sampled measurements used for state-feedback (top plot) and the sampling interval pattern (bottom plot).

The top plot of Figure 6.6 depicts the actuator state dynamics, $\dot{x}_a(t)$, constrained by $\mu_R = 1$, with a zoom-in on the first 10 seconds of the experiment. Since $\dot{x}_a(t)$ is bounded by the function sat_R , the actuator variation is limited at the beginning of the experiment, reaching the rate saturation bounds. The bottom plot shows the $\text{sat}_M(u(t))$ signal alongside the actuator state. Due to the rate saturation, the actuator requires approximately 10 seconds to reach the desired controller signal, which highlights the importance of accounting for saturation limits in the controller design. Despite this constraint, the system remains stable and converges asymptotically to the origin.

The time-varying parameters are presented in Figure 6.7. The top plot shows the evolution of the parameters in continuous time, while the bottom plot depicts the measured parameters, highlighting the parameter mismatch phenomenon inherent to sampled-data control with parameter-dependent controllers.

The same data is collected using an increasing sampling interval pattern from $\mathcal{T}_1 = 1$ s to $\mathcal{T}_2 = 30$ s to demonstrate the efficiency of the designed controller under varying sampling

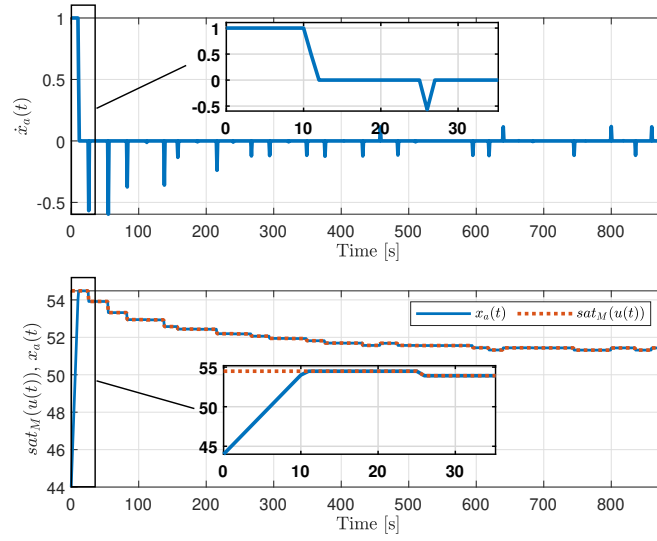


Figure 6.6: Actuator state dynamics $\dot{x}_a(t)$ with rate saturation $\mu_R = 1$ (top) and the corresponding $\text{sat}_M(u(t))$ signal with the actuator state (bottom), with zoom in the first 10 s, for $\eta_k = \mathcal{T}_2 = 30$ s.

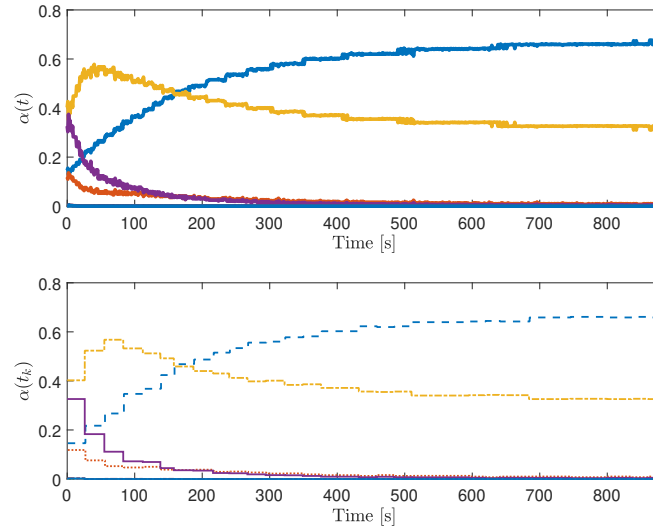


Figure 6.7: Time-varying parameters during the experiment, with $\eta_k = \mathcal{T}_2 = 30$ s. Continuous-time evolution (top plot) and corresponding sampled parameters used for control (bottom plot), illustrating the parameter mismatch inherent to sampled-data control with parameter-dependent controllers.

intervals. The results are presented in Figures 6.8–6.10. Figure 6.8 shows the tank levels and the corresponding sampled measurements used for state-feedback, together with the sampling interval pattern applied during the experiment. Figure 6.9 highlights the actuator dynamics and the effect of the rate saturation, with a zoom on the first 10 seconds of the experiment. Finally, Figure 6.10 presents the evolution of the time-varying parameters in both continuous time and their sampled versions, illustrating the parameter mismatch phenomena.

These experimental results corroborate the discussions presented in Chapter 1 regarding

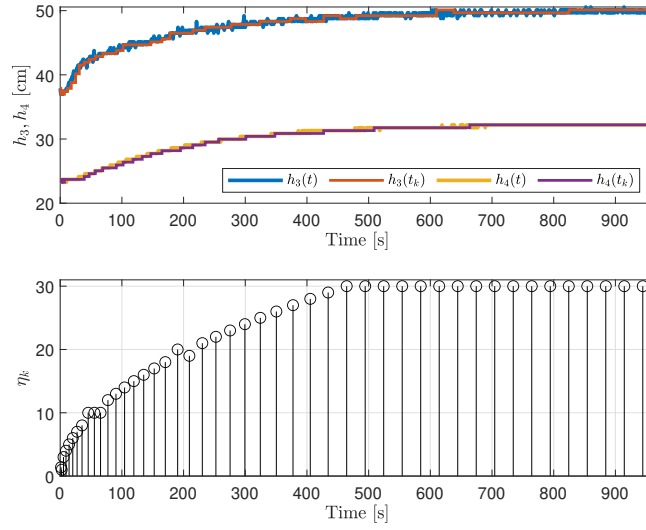


Figure 6.8: Real-time experiment under saturating actuator with increasing sampling interval as $\eta_{k+1} = \eta_k + 1$, $\eta_0 = 1$ s. Tank levels and corresponding sampled measurements used for state-feedback (top plot) and the sampling interval pattern (bottom plot).

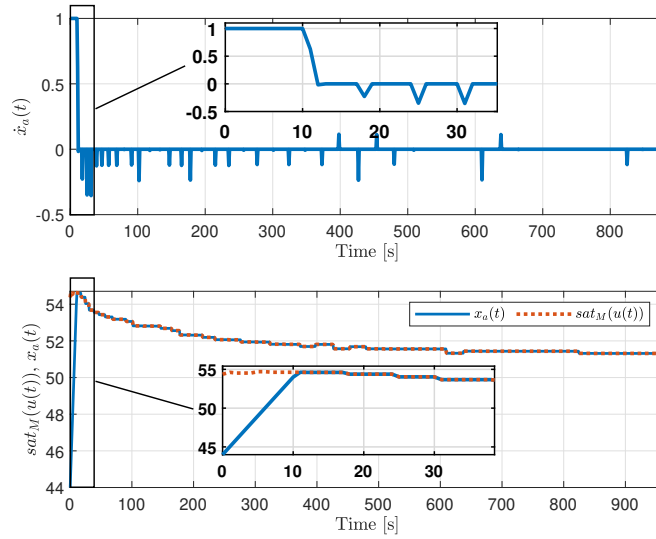


Figure 6.9: Actuator state dynamics $\dot{x}_a(t)$ with rate saturation $\mu_R = 1$ (top) and the corresponding $\text{sat}_M(u(t))$ signal with the actuator state (bottom), with zoom in the first 10 s, for $\eta_{k+1} = \eta_k + 1$, $\eta_0 = 1$ s.

the intrinsic challenges of discretizing nonlinear systems. By relying on the continuous-time framework with sampled-data analysis, the designed controller guarantees stability even under large and irregular sampling intervals, which naturally arise in networked control systems. This highlights the robustness of the proposed approach to practical network-induced imperfections, such as variable transmission delays and packet dropouts. It is important to emphasize that such stabilization would not be feasible within the discrete-time model-based framework discussed in Chapter 3, since the discretization with such a large sampling time does not faithfully represent the continuous-time dynamics and leads to the loss of essential system characteris-

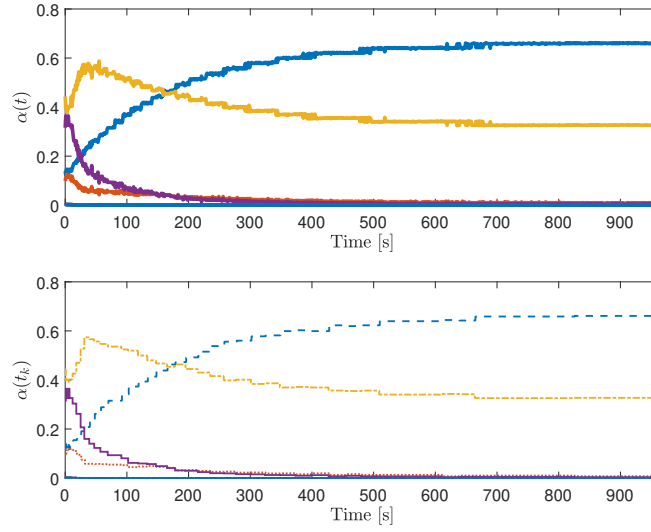


Figure 6.10: Time-varying parameters during the experiment, with $\eta_{k+1} = \eta_k + 1$, $\eta_0 = 1$ s. Continuous-time evolution (top plot) and corresponding sampled parameters used for control (bottom plot), illustrating the parameter mismatch inherent to sampled-data control with parameter-dependent controllers.

tics. In contrast, the sampled-data approach enables the direct design of digital controllers from the continuous-time model, preserving its fundamental properties while accommodating aperiodic sampling. Ultimately, this contrast demonstrates that, while the discrete-time framework of Chapter 3 is inherently constrained by the limitations of discretization, the sampled-data methodology opens the door to stabilization under conditions that would otherwise not be feasible. Hence, the real-time validation not only supports the theoretical developments of this thesis but also demonstrates their practical relevance for nonlinear systems subject to aperiodic samplings. This constitutes one of the main experimental achievements of the thesis: demonstrating that stabilization under such large sampling intervals, impracticable by discrete-time models, becomes possible through the proposed sampled-data framework.

6.6 Final Considerations

This chapter has presented a novel approach for designing parameter-dependent sampled-data state feedback controllers for LPV systems with actuators subject to both magnitude and rate saturation, using LMIs. The method has combined the input-delay approach and a looped-functional, whose derivative has been verified through a second-order polynomial negativity lemma. Optimization procedures were used to enhance the aperiodic sampling interval and maximize the region of attraction, ensuring stable convergence of trajectories to the origin. Two numerical examples have demonstrated the efficiency of the proposed design, showing less conservative stability conditions compared to existing methods. Moreover, experimental validation was performed on a sampled-data control of a nonlinear tank-level system, considering actuator saturation both in magnitude and rate of variation, confirming the practical applicability of the proposed approach.

Conclusion and Perspectives

Sin embargo, antes de llegar al verso final ya había comprendido que no saldría jamás de ese cuarto, pues estaba previsto que la ciudad de los espejos (o los espejismos) sería arrasada por el viento y desterrada de la memoria de los hombres en el instante en que Aureliano Babilonia acabara de descifrar los pergaminos, y que todo lo escrito en ellos era irrepetible desde siempre y para siempre, porque las estirpes condenadas a cien años de soledad no tenían una segunda oportunidad sobre la tierra.

Cien Años de Soledad (1967)
GABRIEL GARCÍA MARQUEZ

In this final chapter, a broad overview of the development and main findings of this thesis is provided. Perspectives for future work are also outlined. In addition, other results conceived during the doctoral research period are briefly discussed, followed by the list of publications derived from this research.

7.1 Concluding Remarks

First of all, this thesis can be regarded as a historical (positive) chronological record of the doctoral research period, structured chronologically to reflect the natural evolution of investigation, methods, and results throughout the research. Each chapter represents a stage of this trajectory: from the formulation of fundamental concepts and methodological tools, through the development of theoretical contributions, and finally to practical applications and collaborations. Presenting the work in this temporal sequence highlights not only the logical organization of the research but also how the different studies were interconnected, progressively converging toward the main objective and contribution.

To better illustrate this perspective, the objectives of the thesis are revisited and summarized in Figure 7.1. The first step was to develop stabilization conditions for LPV systems with actuators subject to rate and magnitude saturation. Although the design of controllers based on discrete-time models is strongly discouraged throughout this thesis due to its limitations for nonlinear systems, exploring this approach in Chapter 3 was necessary to understand the general methodology and limitations for designing controllers that remain stable under actuator

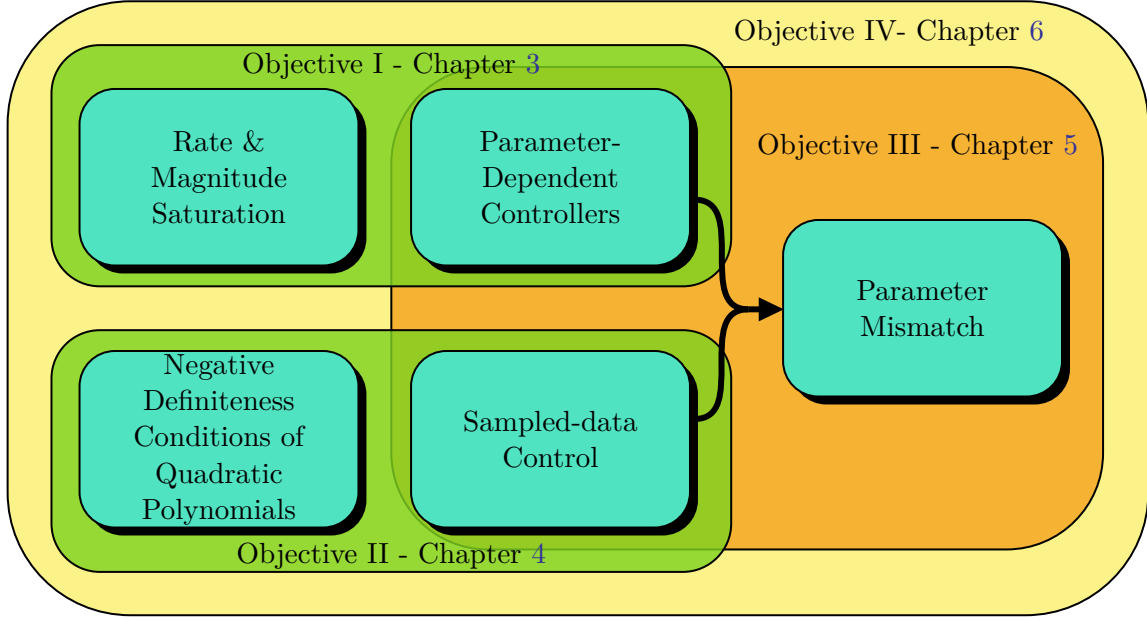


Figure 7.1: Summary of the main topics and objectives of this doctoral thesis.

saturation. The particular case of systems subject to amplitude-bounded disturbances was chosen to address a gap in the literature and to allow fair comparisons with recent works.

From the beginning of this thesis, the unexplored sampled-data control of LPV systems with actuators subject to rate and magnitude saturation was identified as the main objective, particularly to mitigate issues arising from the discretization of nonlinear systems, in contrast to the proposition of Chapter 3, and allowing larger sampling intervals due to network issues. The starting point was the works of [Seuret \(2012\)](#); [Seuret and Gouaisbaut \(2013\)](#); [Palmeira et al. \(2016\)](#) for precisely known systems, as well as the well-explored LPV cases in [Gomes da Silva Jr. et al. \(2015, 2018\)](#); [Palmeira et al. \(2021, 2022\)](#), using the input-delay approach for sample-data control systems proposed by [Fridman \(2010\)](#). From the detailed study of these works, two central aspects emerged as the focus for progressing toward the final objectives. The first concerned the use of looped-functionals whose derivatives result in a quadratic polynomial in the delay that must be ensured to be negative to satisfy the looped stability lemma proposed by [Seuret \(2012\)](#). Unlike the aforementioned works, where the chosen looped-functionals result in a linear polynomial and negative definiteness can be ensured by convexity, the case of second-order functions involves an extensive literature with several methods to guarantee negative definiteness. However, these approaches were not explored in the context of LPV systems, highlighting a gap in the literature addressed in this thesis in Chapter 4 with three novel approaches to address such a problem.

The second main aspect concerns the parameter mismatch induced by parameter-dependent controllers in sampled-data control of LPV systems. This issue is addressed in the works of [Gomes da Silva Jr. et al. \(2015, 2018\)](#); [Palmeira et al. \(2021, 2022\)](#), providing approaches that could be directly implemented in this thesis. However, this aspect was identified as suitable for further investigation to achieve less conservative solutions. Therefore, the mean-value theorem

arose as a potential tool to mitigate the effects of parameter mismatch, as discussed in Chapter 5. In addition, a natural path was to include parameter-dependent looped-functionals whose derivative results in quadratic polynomials to obtain robust and parameter-dependent controllers for LPV systems, as also presented in Chapter 5.

Finally, the final objective of this thesis, integrating the theoretical and methodological developments explored throughout the doctoral research, was accomplished: to tackle the sampled-data control of LPV systems with actuators subject to both magnitude and rate saturation. Building upon the design of parameter-dependent controllers, the application of the mean-value theorem to mitigate parameter mismatches, and the use of quadratic looped-functionals to ensure reduced-conservatism stabilization conditions, the resulting approach eliminates the need for discrete-time plant models and allows larger and aperiodic sampling instants. This methodology addresses the limitations of the conditions presented in Chapter 3, offering a practically applicable solution that guarantees stability under realistic network and actuator constraints. In this way, the thesis demonstrates the sequential exploration of individual contributions, culminating in an integrated framework that effectively addresses the practical challenges of LPV sampled-data control under magnitude and rate saturating actuators.

The main contributions of the chapters of this thesis are summarized in the following:

Chapter 3 has introduced the design of parameter-dependent controllers for discrete-time varying systems with magnitude and rate saturating actuators. The methodology has accommodated LPV, *quasi*-LPV, and T-S fuzzy models, characterizing the set of initial conditions leading to convergence of system trajectories. Parameter-dependent state-feedback gains have ensured local stabilization of the saturated system, while bounded exogenous disturbances have been considered with an estimation of the ℓ_2/ℓ_∞ gain from disturbance to regulated output. Convex optimization has been employed to design stabilizing gains that maximize the region of attraction, allowable disturbance levels, or minimize the gain. The approach has been illustrated with numerical examples and has been validated experimentally on a nonlinear level control system. The drawback of this contribution lies in the hypothesis of regular sampling, which is a basis for conventional discrete-time control. Such an aspect was overcome in the next chapters.

Chapter 4 has proposed new approaches to ensure the negative definiteness of quadratic functions, fundamental for stability analysis and control design of time-varying delayed systems. Quadratic functions have been reformulated as homogeneous polynomials or Bézier curves, enabling less conservative non-monotonic convergent conditions via Young's inequality and monotonic convergent conditions through Pólya's Theorem or de Casteljau's algorithm. Unlike previous partitioning methods, the proposed monotonic convergent conditions have provided guaranteed solutions for sufficiently large Pólya degree or number of de Casteljau iterations. The effectiveness of these approaches has been illustrated on a scalar quadratic example and an LPV sampled-data control benchmark, showing advantages over existing methods.

Chapter 5 has developed a simple yet effective method based on the mean value theorem to tackle parameter mismatches between the controller and the plant during intersampling periods in LPV sampled-data control. A key advantage of the proposed approach has been that it pre-

vents mismatch terms from spreading across the parameter-dependent matrices in the controller synthesis conditions, ensuring that these effects vanish at the sampling instants. This has allowed for more accurate estimates of the maximum admissible sampling interval, outperforming recent methods even when compared to more complex approaches. The methodology further has incorporated a looped-functional whose time derivative results in a quadratic polynomial, with negativity ensured via a Pólya-based lemma, making it suitable for the derivative of the looped-functional in controller design. Additional conditions have been introduced to handle *quasi*-LPV systems, extending the applicability of the method. The effectiveness of the approach has been illustrated through numerical examples and further validated through real-time experiments on a nonlinear tank system with industrial actuators, demonstrating practical feasibility and improvements in the design of sampled-data controllers under realistic constraints.

Chapter 6 has presented a novel approach for designing parameter-dependent sampled-data state feedback controllers for LPV systems with actuators subject to both magnitude and rate saturation. The method has combined the input-delay approach with a looped-functional, whose derivative has been verified through a second-order polynomial negativity lemma. Optimization procedures have been employed to enhance the aperiodic sampling interval and to maximize the region of attraction, ensuring stable convergence of trajectories to the origin. The efficiency and reduced conservatism of the proposed approach have been demonstrated through two numerical examples. Furthermore, experimental validation has been performed on a nonlinear tank-level system under sampled-data control, considering actuator saturation in both magnitude and rate, confirming the practical applicability of the methodology.

7.1.1 Future Research

In this section, some suggestions for potential directions for future research are presented. They are based on the further development and extension of the main contributions of this thesis. The potential research directions are outlined as follows:

- The conditions developed in this thesis can be extended to other conceptual problems in the literature, such as the presence of disturbances limited in energy and amplitude with guaranteed $\mathcal{L}_2/\mathcal{L}_\infty$ cost;
- The methods based on Pólya's theorem and on Bézier curves with the de Casteljau algorithm, proposed in Chapter 4 for negative definiteness conditions of quadratic polynomials, can be extended to polynomials of arbitrary order, thereby broadening their applicability. Furthermore, in all the results of this thesis, Jensen's inequality was employed to bound a term of the proposed looped-functionals. Such a use intends to keep comparable the proposed conditions in this thesis with other works in the literature. The use of Bessel–Legendre inequalities (Seuret and Gouaisbaut, 2018; Bajodek et al., 2024) can be investigated instead, particularly in the case of negative definiteness conditions of arbitrary order, with the potential to yield less conservative solutions;

- By rewriting a homogeneous polynomial as a Bernstein polynomial (4.6), another research direction can be explored to derive necessary and sufficient conditions for stability analysis and controller synthesis of parameter-dependent systems. This is particularly relevant when considering Lyapunov functions of higher degree, providing an alternative to Pólya's theorem (Oliveira and Peres, 2005);
- The mean value theorem, employed to mitigate the effects of parameter mismatch in the synthesis conditions of parameter-dependent sampled-data controllers for LPV systems in Chapter 5, can be further investigated in the context of hybrid or impulsive systems (Briat, 2021). This represents a promising research direction for modeling sampled-data control systems.

7.2 Additional Results Developed Along the Doctoral Research

This section presents other relevant results obtained using the findings of this thesis, which have led to the publication or submission of scientific papers in collaboration with other researchers.

7.2.1 Network Control Systems Design under Deception Attacks with Dynamic Event-Triggering Mechanism

This work, developed in collaboration with researchers from the *20 August University* (Algeria), addresses the design of event-triggered controllers for a class of linear Networked Control Systems (NCS), subject to transmission delays and stochastic deception attacks. The main contribution of the author of this thesis was the introduction of the negative definiteness lemma for quadratic polynomials, discussed in Chapter 4, to obtain less conservative conditions.

With the advances of communication network technologies, there is a preference for networked control systems (NCS) in industrial applications, due to their high efficiency and benefits, low cost, ease of maintenance, and better reliability. NCS also brings control challenges in dealing with network-induced delays, packet dropouts, network bandwidth limitations, and cyber attacks that can compromise closed-loop performance or even lead to instability. Several works in the current literature dealt with such problems, see e.g., Zhang and Han (2014); Li et al. (2021b); Rouamel et al. (2022a, 2021). Furthermore, low-cost implementation of many NCS (low computing and processing capabilities of system components like sensors, control devices), makes it necessary to provide design conditions to prevent such critical issues. Hence, Event-triggered mechanisms (ETM) provide an interesting feature in resource-constrained applications, reducing the number of data samplings and/or packet transmissions, compared to classical time-triggered mechanisms, which in turn allows to mitigate network traffic or congestion and alleviates the mentioned impacting network-induced phenomena to guarantee the quality-of-service required in NCS design (Li et al., 2016; Ge et al., 2021; Rouamel et al., 2021; Hu et al., 2022; Zhang et al., 2022a). One of the main goals of ETM design lies in balancing the control performance with respect to the reduction of the network loads. In many previous studies, a pre-fixed static triggering threshold is considered (see e.g., Shi et al. (2021); Wang et al. (2020); Zhang et al. (2021)), which is not adjusted online regarding the system

and network behavior. Recently, dynamic ETMs were introduced in [Tian et al. \(2019\)](#); [Chen et al. \(2022c\)](#); [Rouamel et al. \(2021, 2022b\)](#), wherein the threshold is self-adjusted according to the system dynamics to further reduce the number of transmitted packets.

Additionally, such a considered NCS framework, involving communication networks, may be subject to cyberattacks. Among the different ways to model them, the focus is put in this paper on NCS affected by stochastic deception attacks (see, e.g., [Wang et al. \(2017, 2020\)](#)). In this context, the attacked closed-loop NCS can be regarded as a dynamical system involving stochastically distributed input time-varying delays. Also, the stability analysis and/or stabilization of such attacked NCS is often made via Lyapunov-Krasovskii Functionals (LKF) to provide Linear Matrix Inequality (LMI) based design conditions. However, such design conditions are usually only sufficient, so they suffer from conservatism. Therefore, relaxation of these conditions can help improve deception attack resilience, as well as reducing the number of packets transmitted by using ETM, which is the goal followed in this study. The control architecture under consideration, which includes the communication network and the points susceptible to deception attacks, is illustrated in Figure 7.2.

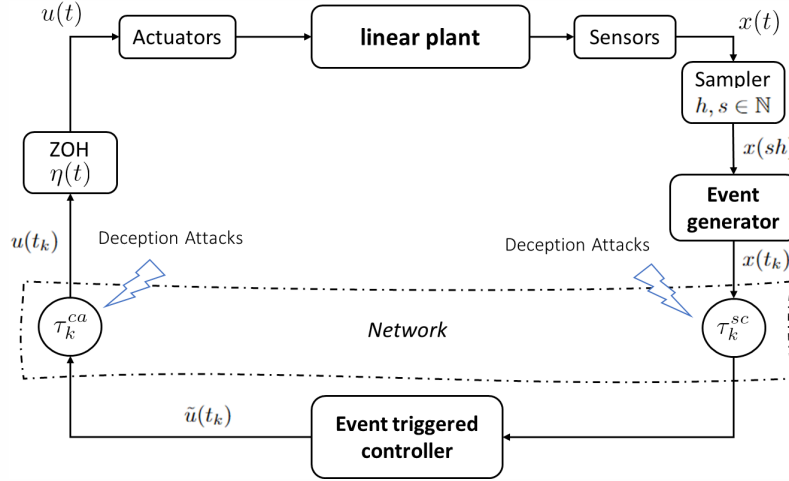


Figure 7.2: Block diagram of the NCS under the proposed dynamic ETM and subject to deception attacks.

The main contribution of this work is to propose an improved dynamic ETM to alleviate the network loads when the system is approaching its equilibrium, together with the design of a stabilizing state feedback networked controller, resilient to deception attacks. In this context, to cope with the security problem of NCSs, randomly occurring deception attacks are considered in both the sensors-to-controller and controller-to-actuators channels, where bounded malicious signals can be injected by attackers. On the basis of the dynamic ETM, new relaxed LMIs-based design conditions are proposed by selecting a suitable LKF.

This work was published in: Rouamel, M., Oliveira, L. A. L., Bourahala, F., Guelton, K., and Motchon, K. M. D. (2023). Network control systems design under deception attacks with dynamic event-triggering mechanism. *IEEE Control Systems Letters*, 7, 3265-3270.

7.2.2 Co-Design and Evolutionary Optimization of Periodic Event-Triggered Robust Control for LPV Systems

This work was developed in collaboration with an undergraduate student from CEFET-MG, whose initial research results indicated promising potential for novel contributions in the field of cyber-physical systems control. This collaboration resulted in a paper accepted at an international conference and fostered the student's interest in pursuing a master's degree on a topic directly related to this work.

Networked control systems (NCSs) are widely used in industrial systems due to Industry 4.0 and the Internet of Things (IoT). Such a framework offers several advantages, including lower operational costs, reduced energy consumption, and architectural flexibility since the control elements can be distributed over digital communication networks, including wireless ones (Hespanha et al., 2007; Zhang et al., 2020b). However, constraints from limited communication resources, including restricted bandwidth and limited energy availability, along with challenges inherent to communication networks, like transmission delays, quantization, and packet dropouts, can compromise the closed-loop stability of the system or lead to significant performance degradation (Peng and Li, 2018).

To ensure closed-loop stability and performance while optimizing network resource utilization, event-triggered control (ETC) has emerged as an effective strategy for balancing network usage and stability criteria (Tabuada, 2007; Hespanha et al., 2007). The ETC operates by transmitting information over the network and updating the control signal only when specific criteria, such as state or output conditions, are satisfied. Such a mechanism prevents unnecessary data transmissions and releases network communications resources for other devices or processes belonging to the NCS. The design of event-triggered mechanisms (ETMs) can follow two paths: emulation (Girard, 2015; Zhang et al., 2020b; Xu et al., 2024) and co-design (Coutinho and Palhares, 2021; de Souza et al., 2022; Peixoto et al., 2024). In emulation, an ETM is designed for an available stabilizing controller. In co-design, on the other hand, both the stabilizing controller and the ETM are designed simultaneously, optimizing the network resources used with guaranteed stability.

A critical aspect of ETM design is selecting the triggering function. Literature categorizes strategies into static (Tabuada, 2007; de Souza et al., 2022; Xu et al., 2024) and dynamic (Girard, 2015; Coutinho and Palhares, 2021; Rouamel et al., 2023; de Souza et al., 2024), all aiming to minimize network transmissions. In general, the design of ETMs parameters finds convex formulations in the literature (Zhang et al., 2020b; de Souza et al., 2024). However, these formulations are indirect optimization procedures and, consequently, do not achieve an optimum tuning of ETM. This limitation has led to exploring alternative approaches, like combining differential evolution (DE) algorithms (Storn and Price, 1997; Gonçalves et al., 2024) with convex methods, to directly optimize ETM performance (Rodrigues et al., 2024).

System measurements in ETCs can be performed in continuous (Tabuada, 2007) or discrete-time (de Souza et al., 2022), according to the plant temporal domain. However, with the advent of sampled-data systems (Fridman, 2010; Seuret, 2012), a mixed framework arises, where the plant evolves in the continuous-time domain while the periodic ETM and the controller are designed in the discrete-time one. Such an alternative is well-suited for digital devices operating through communication networks. Note that plant sensing can be performed either periodically (Heemels et al., 2013) (commonly referred to as Periodic Event-Triggered Control (PETC)) or aperiodically (Xu et al., 2024). In this context, the input-delay approach (Fridman, 2010) can be employed to model the sampling

phenomenon, enabling the use of looped Lyapunov-Krasovskii functionals (LKF) (Seuret, 2012) to derive stability conditions in terms of Linear Matrix Inequalities (LMIs). A notable approach to reduce LKF conservatism involves constructing them to produce a time-derivative that depends polynomially on the delay parameter. Negativity lemmas have been proposed to ensure the negative definiteness of LKFs time-derivative, but the process becomes challenging whenever parameter-dependent systems are involved. Recently, Oliveira et al. (2025a) proposed an effective solution to this challenge by means of Pólya's Theorem.

Despite the extensive literature on ETM and controller co-design and stability conditions for PETCs, there are unsolved issues, particularly concerning sampled-data control of linear parameter-varying (LPV) systems. In Peixoto et al. (2024), co-design conditions for parameter-dependent controllers and ETMs in LPV systems were proposed under denial-of-service and stochastic deception attacks. This approach assumes that the time-varying parameters of the controller evolve continuously, while only the system states are periodically sampled and transmitted based on event-triggered mechanisms. However, this assumption may be unrealistic, requiring the transmission of state and parameters in real-world applications. In Zhang et al. (2020b), the parameter mismatch between the plant and sampled controller is addressed for PETC emulation design in LPV systems. Moreover, the proposed LMI conditions are limited to the emulation approach, relying on a pre-designed controller and not accounting for the co-design of the controller and ETM. The control topology addressed in this paper is depicted in Figure 7.3.

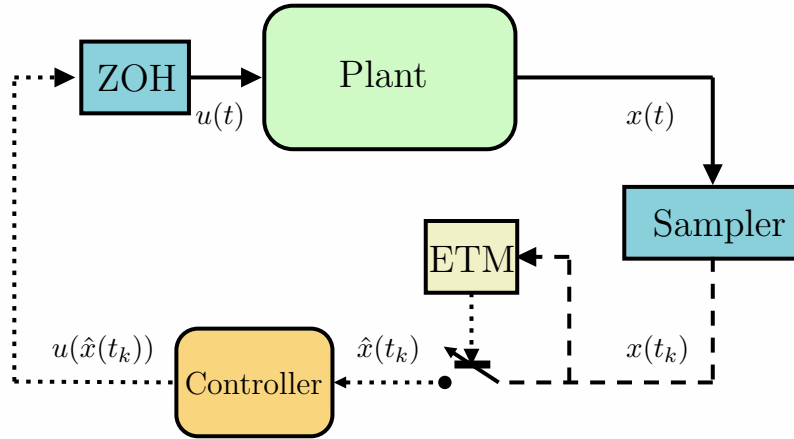


Figure 7.3: Topology of the event-triggered control.

This work focuses on the co-design of PETC with robust controllers for LPV systems, where the plant states and parameters evolve in continuous-time, and the time-varying parameters are unavailable for feedback. The proposed approach employs parameter-dependent looped-functionals, whose derivatives yield second-order polynomials, with negativity ensured via Pólya's theorem. LMI-based conditions are derived to guarantee the asymptotic stability of the closed-loop LPV system under a static ETM that regulates state transmissions. A two-stage optimization procedure using DE algorithms is introduced to minimize control signal updates compared to standard periodic sampling.

This work was published in: Oliveira, L. A. L., Rodrigues, M. L., Guelton, K., Motchon, K. M. D., and Leite, V. J. S. (2025). Co-design and evolutionary optimization of periodic event-triggered robust control for LPV systems. In *Proceedings of the 11th IFAC Symposium on Robust Control Design (ROCOND'25)*, Porto, Portugal.

7.2.3 Relaxed Co-Design Conditions for Periodic Event-Triggered Control of Lur'e Systems

This work was developed in collaboration with researchers from the *Federal University of Rio Grande do Sul* (UFRGS), who identified that the looped-functional proposed in Chapter 4 could lead to less conservative results in their preliminary conditions.

Periodic Event-Triggered Control (PETC) was introduced by Heemels et al. (2013) as an alternative method for dealing with networked control systems. These systems have become increasingly widespread in industry in recent years due to their ease of installation, architectural flexibility, and reduced cost (Zhang et al., 2020b). The central idea is to incorporate an Event-Triggering Mechanism (ETM) into the control loop, which periodically monitors system variables and updates the control signal only when necessary to ensure stability and performance. The periodic implementation is more suitable for digital platforms compared to continuously monitored ETMs, as it does not require dedicated hardware (Tabuada, 2007) and implicitly avoids Zeno behavior (Goebel et al., 2012). However, many studies neglect the system dynamics between sampling instants in the stability analysis (Heemels and Donkers, 2013). In this case, the main challenge lies in formulating stability conditions that explicitly account for the continuous evolution of the plant states, the periodic monitoring of the ETM, and the aperiodic updates of the control signal (events).

Regarding the design of PETC techniques, the literature identifies two main paradigms: emulation and co-design. In the emulation approach, a stabilizing controller is given a priori, and the objective is to compute the ETM in a way that guarantees closed-loop stability (Tabuada, 2007). In co-design, the controller and the ETM are designed simultaneously. This constitutes an emerging research topic for nonlinear systems and is more challenging than emulation, as it requires the proper handling of conflicting constraints between control specifications and event reduction (Coutinho et al., 2024).

In this context, the selection of a suitable ETM is crucial for managing the trade-off between stability, performance, and event reduction. Tabuada (2007) proposed a static ETM, in which an event occurs only if the current state deviates beyond a fixed threshold from its value at the previous event. To further reduce the number of events, Girard (2015) introduced a dynamic threshold in the triggering rule, capable of ensuring the same level of performance achieved with the static ETM, but with fewer (potential) triggers. However, due to the challenge of certifying closed-loop stability, few studies address PETC strategies with dynamic ETMs, with exceptions such as Coutinho et al. (2021); Lisbôa et al. (2024). None of these works provides stability conditions for Lur'e systems following a co-design approach. The control topology addressed in this paper is depicted in Figure 7.4.

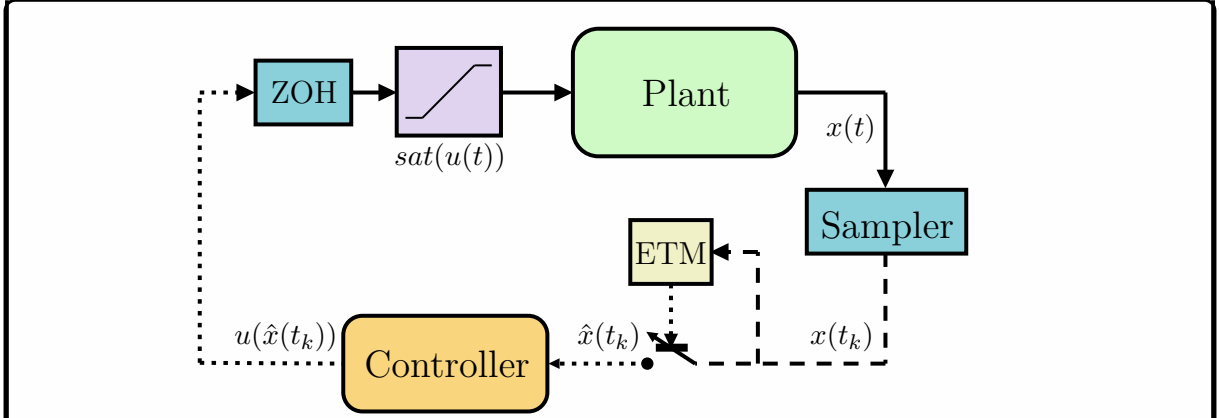


Figure 7.4: Topology of the event-triggered control for Lur'e systems.

This work investigates the co-design of a state-feedback control law with PETC and a dynamic ETM for continuous-time Lur'e systems. Based on Lyapunov arguments, the looped-functional technique, and quadratic polynomial inequalities, constructive conditions are derived in the form of linear matrix inequalities (LMIs) to achieve global stabilization of the system origin in closed loop. Unlike [Lisbôa et al. \(2024\)](#), a more general functional is employed, following the approach in [Oliveira et al. \(2025a\)](#), with quadratic dependence on the delay variable. These conditions are incorporated into a convex optimization procedure formulated to systematically compute the gains of the control law and the parameters of the dynamic ETM. The objective is to reduce the number of events compared to the time-triggered strategy, in which the control signal is updated at every monitoring instant.

This work was published in: Lisbôa, C., Flores, J. V., Moreira, L. G., **Oliveira, L. A. L.**, and Leite, V. J. S. (2025). Condições relaxadas de co-design em controle periódico disparado por eventos para sistemas Lur'e. In *Anais do XVII Simpósio Brasileiro de Automação Inteligente*, São João del Rei-MG, Brazil.

7.3 Publications

All publications of the author linked with the doctoral research are listed in the following:

Journal papers:

- a) **Oliveira, L. A. L.**, Barbosa, M. V. C., Silva, L. F. P., Leite, V. J. S. (2021), Exponential stabilization of LPV systems under magnitude and rate saturating actuators, *IEEE Control Systems Letters*, 6, 1418-1423
- b) Rouamel, M., **Oliveira, L. A. L.**, Bourahala, F., Guelton, K., and Motchon, K. M. D. (2023). Network control systems design under deception attacks with dynamic event-triggering mechanism. *IEEE Control Systems Letters*, 7, 3265-3270;
- c) **Oliveira, L. A. L.**, Leite, V. J. S., Silva, L. F. P., and Guelton, K. (2024). Stabilization of polytopic discrete-time varying systems with magnitude and rate saturating actuators and amplitude bounded disturbances. *Journal of the Franklin Institute*, 361(10), 106915;

- d) **Oliveira, L. A. L.**, Guelton, K., Motchon, K. M. D., and Leite, V. J. S. (2025). New negative definiteness conditions for quadratic functions with illustration in LPV sampled-data control. *Automatica*, 173, 112077.
- e) **Oliveira, L. A. L.**, Guelton, K., Motchon, K. M. D., and Leite, V. J. S. (2025). Refining parameter mismatch handling in LPV sampled-data controller design. *Automatica*, 180, 112470.
- f) **Oliveira, L. A. L.**, Guelton, K., Motchon, K. M. D., and Leite, V. J. S. (2025). Sampled-data control of LPV systems with magnitude and rate saturating actuators. *IEEE Control Systems Letters*, 9, 1165-1170.
- g) **Oliveira, L. A. L.**, Barbosa, M. V. C., Motchon, K. M. D., Guelton, K., and Leite, V. J. S. (2026). Robust Stabilization of LPV Sampled-Data Systems via Quadratic Polynomial Conditions. *Journal of Control, Automation and Electrical Systems*.

Conference papers:

- a) **Oliveira, L. A. L.**, Leite, V. J. S., Silva, L. F. P., and Guelton, K. (2022). Robust regional input-to-state stabilization of discrete-time systems under magnitude and rate saturating actuators. In *Anais do XXIV Congresso Brasileiro de Automática*, Fortaleza-CE, Brazil.
- b) **Oliveira, L. A. L.**, Guelton, K., Motchon, K. M. D., and Leite, V. J. S. (2023). Conditions asymptotiquement nécessaires et suffisantes pour des inégalités polynomiales matricielles du second ordre. In *1er Congrès Annuel de la SAGIP*, Marseille, France.
- c) **Oliveira, L. A. L.**, Guelton, K., Motchon, K. M. D., and Leite, V. J. S. (2024). Estabilização robusta via realimentação de estados amostrados em sistemas LPV: condições polinomiais quadráticas. In *Anais do XXV Congresso Brasileiro de Automática*, Rio de Janeiro-RJ, Brazil.
- d) **Oliveira, L. A. L.**, Guelton, K., Motchon, K. M. D., and Leite, V. J. S. (2025). Une approche basée sur le théorème de la valeur moyenne pour la commande échantillonnée à pas variable des systèmes LPV. In *3ème Congrès Annuel de la SAGIP*, Mulhouse, France.
- e) **Oliveira, L. A. L.**, Rodrigues, M. L., Guelton, K., Motchon, K. M. D., and Leite, V. J. S. (2025). Co-design and evolutionary optimization of periodic event-triggered robust control for LPV systems. In *Proceedings of the 11th IFAC Symposium on Robust Control Design (ROCOND'25)*, Porto, Portugal.
- f) Nogueira, G. A., Lopes, A. N. D., **Oliveira, L. A. L.**, and Motchon, K. M. D. (2025). Comparative study between digital controllers and sampled-data approaches for the stabilization of nonlinear systems. In *2025 IEEE International Conference on Fuzzy Systems (FUZZ)*, Reims, France.

- g) **Oliveira, L. A. L.**, Motchon, K. M. D., Guelton, K., and Leite, V. J. S. (2025). Controle robusto via dados amostrados de sistemas LPV com saturação nos atuadores utilizando condições polinomiais quadráticas. In *Anais do XVII Simpósio Brasileiro de Automação Inteligente*, São João del Rei-MG, Brazil.
- h) Campos, A. R. C. R., **Oliveira, L. A. L.**, and Leite, V. J. S. (2025). Projeto de controlador com custo garantido- $\mathcal{L}_2$ para sistemas LPV amostrados. In *Anais do XVII Simpósio Brasileiro de Automação Inteligente*, São João del Rei-MG, Brazil.
- i) Lisbôa, C., Flores, J. V., Moreira, L. G., **Oliveira, L. A. L.**, and Leite, V. J. S. (2025). Condições relaxadas de co-design em controle periódico disparado por eventos para sistemas Lur'e. In *Anais do XVII Simpósio Brasileiro de Automação Inteligente*, São João del Rei-MG, Brazil.
- j) Chibani, A., **Oliveira, L. A. L.**, Motchon, K. M. D., Guelton, K., and Leite, V. J. S. (2025). Robust Periodic Sampled-Data Controller Design for T-S Fuzzy Systems Affected by Low-Frequency Disturbances. In *1st IFAC Joint Conference on Computers, Cognition, and Communication*, Padova, Italy.

Book chapter (in press):

- a) **Oliveira, L. A. L.**, Rodrigues, M. L., Silva, L. F. P., and Leite, V. J. S. Event Trigger Mechanism Design: A Direct Transmission Minimization Strategy.

7.4 Awards

The author of this thesis received the award for best paper in the doctorate category in the *XXV Congresso Brasileiro de Automática (CBA 2024)*, which took place in Rio de Janeiro-RJ, Brazil.



Figure 7.5: Plaque of recognition for the SBA Award – Doctorate 2024, granted by the Brazilian Society of Automatics (SBA) during the XXV Congresso Brasileiro de Automática (CBA 2024) for the best student paper in the doctorate category.

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Appendix A

Equivalent inequality conditions to the statement of Theorem 4.1

Notice that $(\theta_{1i} + \theta_{2i})^d = \sum_{k=0}^d \mathcal{C}_d^k \theta_{1i}^{d-k} \theta_{2i}^k$. Hence, similarly to the method presented in Oliveira and Peres (2005), (4.5) can be rewritten as:

$$\begin{aligned} & \left(\theta_{1i}^2 \mathcal{P}(\bar{\tau}_i) + \theta_{1i} \theta_{2i} \mathcal{F}(\underline{\tau}_i, \bar{\tau}_i) + \theta_{2i}^2 \mathcal{P}(\underline{\tau}_i) \right) \sum_{k=0}^d \mathcal{C}_d^k \theta_{1i}^{d-k} \theta_{2i}^k \\ &= \sum_{k=0}^d \mathcal{C}_d^k \left(\theta_{1i}^{d-k+2} \theta_{2i}^k \mathcal{P}(\bar{\tau}_i) \right. \\ & \quad \left. + \theta_{1i}^{d-k+1} \theta_{2i}^{k+1} \mathcal{F}(\underline{\tau}_i, \bar{\tau}_i) + \theta_{1i}^{d-k} \theta_{2i}^{k+2} \mathcal{P}(\underline{\tau}_i) \right). \end{aligned} \quad (\text{A.1})$$

Let, for $j \in \mathbb{Z}$ and $|j| \leq d \in \mathbb{N}$:

$$\mathcal{C}_d^j = \begin{cases} \mathcal{C}_d^j = \frac{d!}{j!(d-j)!}, & \text{if } j \geq 0, \\ 0, & \text{otherwise.} \end{cases} \quad (\text{A.2})$$

Let $\mathcal{C}_d^k = \mathcal{C}_d^{d-k}$, $\mathcal{C}_d^0 = 1$. Therefore, rearranging the homogeneous polynomial inequality (A.1) from the power symmetry of its monomials, it can be rewritten as:

$$\begin{aligned} & \sum_{k=0}^{\beta_d} \theta_{1i}^{d+2-k} \theta_{2i}^k (\mathcal{C}_d^k \mathcal{P}(\bar{\tau}_i) + \mathcal{C}_d^{k-1} \mathcal{F}(\underline{\tau}_i, \bar{\tau}_i) + \mathcal{C}_d^{k-2} \mathcal{P}(\underline{\tau}_i)) + \\ & \mathbf{1}_{\mathbb{N}(\frac{d}{2})} \theta_{1i}^{\lfloor \frac{d}{2} \rfloor + 1} \theta_{2i}^{\lfloor \frac{d}{2} \rfloor + 1} (\mathcal{C}_d^{\lfloor \frac{d}{2} \rfloor - 1} \mathcal{P}(\bar{\tau}_i) + \mathcal{C}_d^{\lfloor \frac{d}{2} \rfloor} \mathcal{F}(\underline{\tau}_i, \bar{\tau}_i) + \mathcal{C}_d^{\lfloor \frac{d}{2} \rfloor - 1} \mathcal{P}(\underline{\tau}_i)) \\ & + \sum_{k=0}^{\beta_d} \theta_{1i}^k \theta_{2i}^{d+2-k} (\mathcal{C}_d^{k-2} \mathcal{P}(\bar{\tau}_i) + \mathcal{C}_d^{k-1} \mathcal{F}(\underline{\tau}_i, \bar{\tau}_i) + \mathcal{C}_d^k \mathcal{P}(\underline{\tau}_i)). \end{aligned} \quad (\text{A.3})$$

Finally, all the monomials being negative in (A.3), the inequality (2.37) holds if, $\forall(i, k) \in \mathbb{I}_N \times \mathbb{I}_{\beta_d}$:

$$\mathcal{C}_d^k \mathcal{P}(\bar{\tau}_i) + \mathcal{C}_d^{k-1} \mathcal{F}(\underline{\tau}_i, \bar{\tau}_i) + \mathcal{C}_d^{k-2} \mathcal{P}(\underline{\tau}_i) < 0, \quad (\text{A.4})$$

$$\mathcal{C}_d^{k-2} \mathcal{P}(\bar{\tau}_i) + \mathcal{C}_d^{k-1} \mathcal{F}(\underline{\tau}_i, \bar{\tau}_i) + \mathcal{C}_d^k \mathcal{P}(\underline{\tau}_i) < 0, \quad (\text{A.5})$$

and, if $\mathbf{1}_{\mathbb{N}(\frac{d}{2})} = 1$ (d is even or 0):

$$\mathcal{C}_d^{\lfloor \frac{d}{2} \rfloor - 1} \mathcal{P}(\bar{\tau}_i) + \mathcal{C}_d^{\lfloor \frac{d}{2} \rfloor} \mathcal{F}(\underline{\tau}_i, \bar{\tau}_i) + \mathcal{C}_d^{\lfloor \frac{d}{2} \rfloor - 1} \mathcal{P}(\underline{\tau}_i) < 0 \quad (\text{A.6})$$

are satisfied with $\beta_d = \lfloor \frac{d-1}{2} \rfloor + 1$. Hence, (A.4), (A.5) and (A.6) provide equivalent inequality constraints to the statement of Theorem 4.2 with (4.5).

Contrôle échantillonné des systèmes LPV avec actionneurs soumis à saturation en amplitude et en vitesse

Cette thèse étudie la conception de contrôleurs à échantillonnage pour les systèmes à paramètres variant linéairement (LPV) avec des actionneurs soumis à des saturations en amplitude et en vitesse. L'objectif est de garantir la stabilité tout en permettant des intervalles d'échantillonnage admissibles plus larges. La présence croissante de systèmes autonomes et en réseau motive le développement de méthodes de contrôle capables d'intégrer efficacement les processus en temps continu avec les contrôleurs numériques, même sous des contraintes pratiques telles que les délais de communication, les pertes de paquets et la congestion du réseau. Le contrôle à échantillonnage offre un cadre permettant de gérer l'interaction entre les dynamiques continues et les actions discrètes, assurant ainsi à la fois stabilité et performance. Dans les systèmes LPV, des défis supplémentaires apparaissent car le procédé évolue en continu tandis que le contrôleur agit à des instants discrets. Ce décalage peut entraîner des mesures de paramètres asynchrones et un conservatisme potentiel. Cette thèse propose des méthodes pour surmonter les limitations pratiques du contrôle LPV à échantillonnage. L'accent est mis sur l'atténuation des effets néfastes de la saturation des actionneurs et sur la réduction du conservatisme de conception en utilisant la théorie de Lyapunov et des conditions basées sur les inégalités matricielles linéaires (LMI). Premièrement, une approche basée sur un modèle en temps discret est développée pour concevoir des contrôleurs à retour d'état dépendant des paramètres, garantissant la stabilité régionale en présence de saturation des actionneurs et de perturbations limitées en amplitude. Des simulations et des expériences en temps réel sur des systèmes de cuves non linéaires couplées valident cette approche de conception. Deuxièmement, de nouvelles conditions pour la négativité définie des fonctions quadratiques sont introduites en utilisant l'inégalité de Young, le théorème de Pólya et les représentations de Bézier/de Casteljau. Ces conditions garantissent une convergence monotone récursive, produisent des résultats moins conservatifs et sont utilisées pour assurer la négativité de la dérivée temporelle de la fonction de Lyapunov candidate. Enfin, le problème de décalage entre les paramètres du procédé en temps continu et le signal de commande, dépendant des mesures échantillonnées, est traité par une nouvelle application du théorème de la valeur moyenne. Cette approche permet des intervalles d'échantillonnage plus larges. Le cadre est ensuite étendu au contrôle LPV à échantillonnage aperiodique avec des actionneurs saturés. Cette extension évite le recours à des modèles en temps discret du procédé et garantit la stabilité locale grâce à des fonctionnels bouclés dépendant des paramètres et à des conditions de secteur généralisées. Des exemples numériques et des expériences de contrôle en temps réel démontrent la faisabilité et l'efficacité, établissant de nouvelles approches pour le contrôle à échantillonnage des systèmes LPV soumis à des contraintes réalistes d'actionneurs et de réseau.

Systèmes LPV, Contrôle échantillonné, Saturation des actionneurs, Décalage des paramètres, Négativité de polynômes.

Controle amostrado de sistemas LPV com atuadores sujeitos à saturação de magnitude e velocidade

Esta tese investiga o projeto de controladores de dados amostrados para sistemas Linear Parameter Varying (LPV) com atuadores sujeitos à saturação de magnitude e de taxa. O objetivo é garantir a estabilidade ao mesmo tempo em que se permitem intervalos de amostragem admissíveis mais amplos. A presença crescente de sistemas autônomos e em rede motiva o desenvolvimento de métodos de controle capazes de integrar de forma eficiente processos em tempo contínuo com controladores digitais, mesmo sob restrições práticas como atrasos de comunicação, perdas de pacotes e congestionamento de rede. O controle de dados amostrados fornece uma estrutura para lidar com a interação entre dinâmicas em tempo contínuo e ações em tempo discreto, garantindo tanto estabilidade quanto desempenho. Em sistemas LPV, desafios adicionais surgem porque a planta evolui continuamente enquanto o controlador atua em instantes discretos. Esse descompasso pode causar medições assíncronas de parâmetros e potencial conservadorismo. Esta tese propõe métodos para superar limitações práticas no controle LPV de dados amostrados. O foco está na mitigação dos efeitos adversos da saturação dos atuadores e na redução do conservadorismo de projeto por meio da teoria de Lyapunov e de condições baseadas em desigualdades matriciais lineares (LMI). Primeiramente, é desenvolvida uma abordagem baseada em modelo em tempo discreto para projetar controladores de realimentação de estados dependentes dos parâmetros, garantindo estabilidade regional sob saturação dos atuadores e perturbações limitadas em amplitude. Simulações e experimentos em tempo real em sistemas não lineares de tanques acoplados validam essa abordagem de projeto. Em seguida, novas condições para a definitude negativa de funções quadráticas são introduzidas utilizando a desigualdade de Young, o teorema de Pólya e as representações de Bézier/de Casteljau. Essas condições garantem convergência monótona recursiva, produzem resultados menos conservadores e são empregadas para assegurar a negatividade da derivada temporal da função candidata de Lyapunov. Por fim, o descompasso entre os parâmetros da planta em tempo contínuo e o sinal de controle, que depende de medições amostradas, é tratado por meio de uma aplicação inédita do teorema do valor médio. Essa abordagem possibilita intervalos de amostragem mais amplos. O arcabouço é ainda estendido ao controle LPV de dados amostrados aperiódicos com atuadores saturados. Essa extensão evita a necessidade de modelos em tempo discreto da planta e assegura estabilidade local por meio de funcionais em malha dependentes dos parâmetros e de condições setoriais generalizadas. Exemplos numéricos e experimentos de controle em tempo real demonstram a viabilidade e a eficácia, estabelecendo novas abordagens para o controle de dados amostrados de sistemas LPV sob restrições realistas de atuadores e de rede.

Sistemas LPV, Controle amostrado, Saturação de atuadores, Descasamento de parâmetros, Negatividade de polinômios.

This thesis investigates the design of sampled-data controllers for Linear Parameter Varying (LPV) systems with actuators subject to magnitude and rate saturation. The objective is to guarantee stability while allowing larger admissible sampling intervals. The growing presence of autonomous and networked systems motivates the development of control methods that efficiently integrate continuous-time processes with digital controllers, even under practical constraints such as communication delays, packet losses, and network congestion. Sampled-data control provides a framework for handling the interplay between continuous-time dynamics and discrete-time actions, enabling both stability and performance guarantees. In LPV systems, additional challenges arise because the plant evolves continuously while the controller acts at discrete instants. This mismatch may cause asynchronous parameter measurements and potential conservatism. This thesis proposes methods to overcome practical limitations in LPV sampled-data control. The focus is on mitigating the adverse effects of actuator saturation and reducing design conservatism using Lyapunov theory and linear matrix inequality (LMI)-based conditions. First, a discrete-time model-based approach is developed to design parameter-dependent state-feedback controllers, ensuring regional stability under actuator saturation and amplitude-limited disturbances. Simulations and real-time experiments on nonlinear coupled tank systems validate this design approach. Second, new conditions for the negative definiteness of quadratic functions are introduced using Young's inequality, Pólya's theorem, and Bézier/de Casteljau representations. These conditions guarantee recursive monotonic convergence, yield less conservative results, and are employed to ensure the negativity of the time derivative of the Lyapunov candidate function. Finally, the parameter mismatch between continuous-time plant parameters and the control signal, which depends on sampled measurements, is addressed through a novel application of the mean value theorem. This approach enables larger sampling intervals. The framework is further extended to aperiodic sampled-data control of LPV systems with saturated actuators. This extension avoids the need for discretization of plant models and ensures local stability through parameter-dependent looped-functionals and generalized sector conditions. Numerical examples and real-time control experiments show feasibility and effectiveness, establishing new approaches for sampled-data control of LPV systems under realistic actuator and network constraints.

LPV systems, Sampled-data control, Actuator saturation, Parameter mismatch, Polynomial negative definiteness.

Linha de Pesquisa : Sistemas Inteligentes (Brasil)

Spécialité : Automatique (France)

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